A RATING-BASED MODEL FOR CREDIT DERIVATIVES

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We present a model in which a bond B subject to possible default is assessed a ȷcontinuousȷ rating $R_B(t) \in [0, 1)$ that follows a diœusion process, possibly with jumps. Default occurs when the rating reaches 0, which is absorbing. Non-defaultable bonds have rating 1 (unreachable). The value of the bond is the sum of that of its payments. For a given evaluation time t and a given maturity T, the yield to maturity y of a zero-coupon bond depends on the rating:

$$y(t, T, r) = y_1(t, T) + \int_r^1 \varphi(t, T, s)\, ds$$

The non-default yield $y_1(t, T)$ follows a traditional interest rate model (e.g. HJM, BGM, ȷstringȷ, etc.). The ȷspread per unit of ratingȷ $\varphi(t, T, s)$ is a positive random øeld with respect to s and T, e.g. the square of a Gaussian øeld. It is constrained by the fact that $y(t, T, 0)$ is the recovery rate of the zero-coupon bond (possibly itself stochastic). The random øeld φ can be approximated by a ønite dimensional øeld through truncation of the Fourier series of $\sqrt{\varphi}$. In this context, the random øeld φ being given, we compute a risk-neutral drift of the rating process $R_B(t)$, assuming its volatility is also given.

For several bonds, ratings are driven by correlated Wiener processes. In the case of pure diœusion processes, joint defaults have zero probability (though a default occurrence increases the intensity of others). Correlated jumps induce positive joint default probabilities. Credit derivatives are priced by Monte-Carlo simulation: non-standard credit default swaps (CDS), ørst n to default in a basket, etc. Hedge ratios are computed with respect to underlying bonds and CDS's.

Major credit models (Merton, Jarrow-Turnbull, DuŒe-Singleton, Hull-White) are particular cases of this model, which has been specially designed to ease calibration. Long-term statistics on yield spreads in each rating category provide the volatility and correlation structure of the random øeld φ. The rating process is, in a ørst step, statistically estimated, thanks to rating migration statistics from rating agencies (each agency rating is associated with a range for the continuous rating). Then its drift is replaced by the risk-neutral value, while the historical volatility is kept. Jumps are only introduced to model catastrophic events involving several bonds. The rating process being an abstract version of Merton's ørm value, we suggest to use issuers' stock correlation for that of their rating processes (although this hypothesis should be tested).

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