Homework # 2: Mathematics of Finance 1

Due in 3 weeks.

Problem 1. The **Vega** of an option is defined as the sensitivity of the option premium with respect to volatility changes. Derive the theoretical formula for Vega $= \partial C(S, K, T)/\partial\sigma$ in terms of the option parameters σ,r and q. What is your interpretation of Vega as a hedger of option portfolios ? Calculate the Vega of an at-the-money option with one month to expiration with 15% volatility in an interest-rate environment of 6% with $q = 0$. What is the Vega at two months and six months? Another question: what do you think is the use of $\partial\Delta/\partial\sigma$? Give some examples.

Problem 2. Implied volatility of European calls.

(i) Assume that $f(x)$ is a smooth function with $f'(x) > 0$. Show that the equation $f(x) = y$ (y given) can be solved recursively by finding the limit of the sequence x_n defined by

$$x_{n+1} = x_n - \frac{f(x_n) - y}{f'(x_n)} .$$

This is called **Newton's method**.

(ii) Use Newton's method for calculating the implied volatility of European calls. Work out the following examples:

a) Call on S&P 500 index (SPX option of CBOE). Assume an interest rate of 6%, a dividend rate of 2.5% and an index value of 590. Calculate the implied volatility of an option with strike price 600 that trades at $ 10.

b) Dollar-Mark option. Assuming an exchange rate of 1.39 DEM (Deutschemarks) per U.S. dollar, a six-month dollar deposit rate of 6% and a 6 month DEM rate of 4%, suppose that you call a dealer and ask for a market in a six month **dollar put**. The option allows you to exchange $ 1,000,000.00 into DEM at 1.48 DEM per dollar six months from now. The dealer charges $ 85,000.00 for the option. What is the implied volatility? If he is willing to *buy* such an option from you at $ 81,500.00, what is his bid-side volatility?

Problem 3. Portfolio management. On Aug 10, 1993, the S&P 500 index was trading at $ 449.67. Assume you had the following position

Long 40 calls with strike price 455 expiring on Aug 21, 1993 (Price= $ 1)

Long 100 calls with strike price 460 expiring on Aug 21, 1993 (Price = $ 0.3125)

Short 100 puts with strike price 385 expiring on Sept 18, 1993 (Price = 0.1875)

Assume that the interest rate is 7%, the dividend rate is 2.5% and that the times to expiration are respectively 0.036 and 0.112 yr., for the August and September maturities.

(i) Calculate the implied volatilities of the options.

(ii) Calculate the overall Delta of the portfolio.

(iii) Calculate the overall Gamma of the portfolio.

(iv) Construct a hedge for the position assuming that you can buy the stocks in the index for \$ 449.67.

(v) Assume that you hedge and hold the position for 5 days. On that date the index was worth \$ 454.00. Estimate the gains/losses from your strategy, assuming that the options could be bought/sold at their model value on that date.