Information Theory and Financial Modeling, Fall 1998

Homework 1

1. (Buchen & Kelly). Find the minimum-relative-entropy probability density function of a stock price in 100 days assuming that:

Spot price today = 76.875

Interest rate = 5.5%

100 days 70 call, vol =45%

100 days 75 call, vol =45%

100 days 80 call, vol =46%

100 days 85 call, vol =46.5%

100 days 90 call, vol =46%

(Neglect dividend payouts). Assume a lognormal risk-neutral prior distribution with vol=45%.

Price a 100 day European digital call with strike 80. (Derivative security that pays $1 if stock is above 80 at the expiration date and zero otherwise). Extra credit: compute the sensitivities of the price of the digital to the option prices.

2. (Application of Merton & Bielecki-Pliska). Using the daily closing prices of the S&P 500 index of the last 3 years, simulate trading strategies investing in the index and/or cash. (Interest rate=5%). Implement maximum-growth (log utility) or "risk-sensitive" (power utility) strategies. These simulations should include a method for computing μ and σ from the data, using trailing averages or other methods. Compare the results (profit and loss) assuming

log utility and power utility with different exponents

same as above, with no short-selling

log and power, with no borrowing cash

Explain which strategy (i.e. exponent) you would use in practice, if any.