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Some remarks on VIX futures and ETNs

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The ETF Revolution:
From stock baskets to commodities & beyond

• Gold (GLD)	physical (storage)
• Crude Oil WTI (USO)	synthetic (futures)
• Agriculture (DBA)	synthetic (futures)
• Diversified Commos (DBC)	synthetic (futures)
• US Bonds (TLT)	physical (T-bonds)
• Currencies (BZF, FXE, FXB, UUP)	synthetic (swaps)
• VIX Volatility Index (VXX, VXZ, XIV, UVXY, SVXY)	synthetic (futures, notes)
• VSTOXX (EVIX, EXIV)	synthetic (futures, notes)

The CBOE S&P500 Implied Volatility Index (VIX)

- Inspired by Variance Swap Volatility (Whaley, 90's)

$$\sigma_T^2 = \frac{2e^{rT}}{T}\int_0^{\infty} OTM(K,T,S)\frac{dK}{K^2}$$

- Here $OTM(K,T,S)$ represents the value of the OTM (forward) option with strike K, or ATM if S=F.

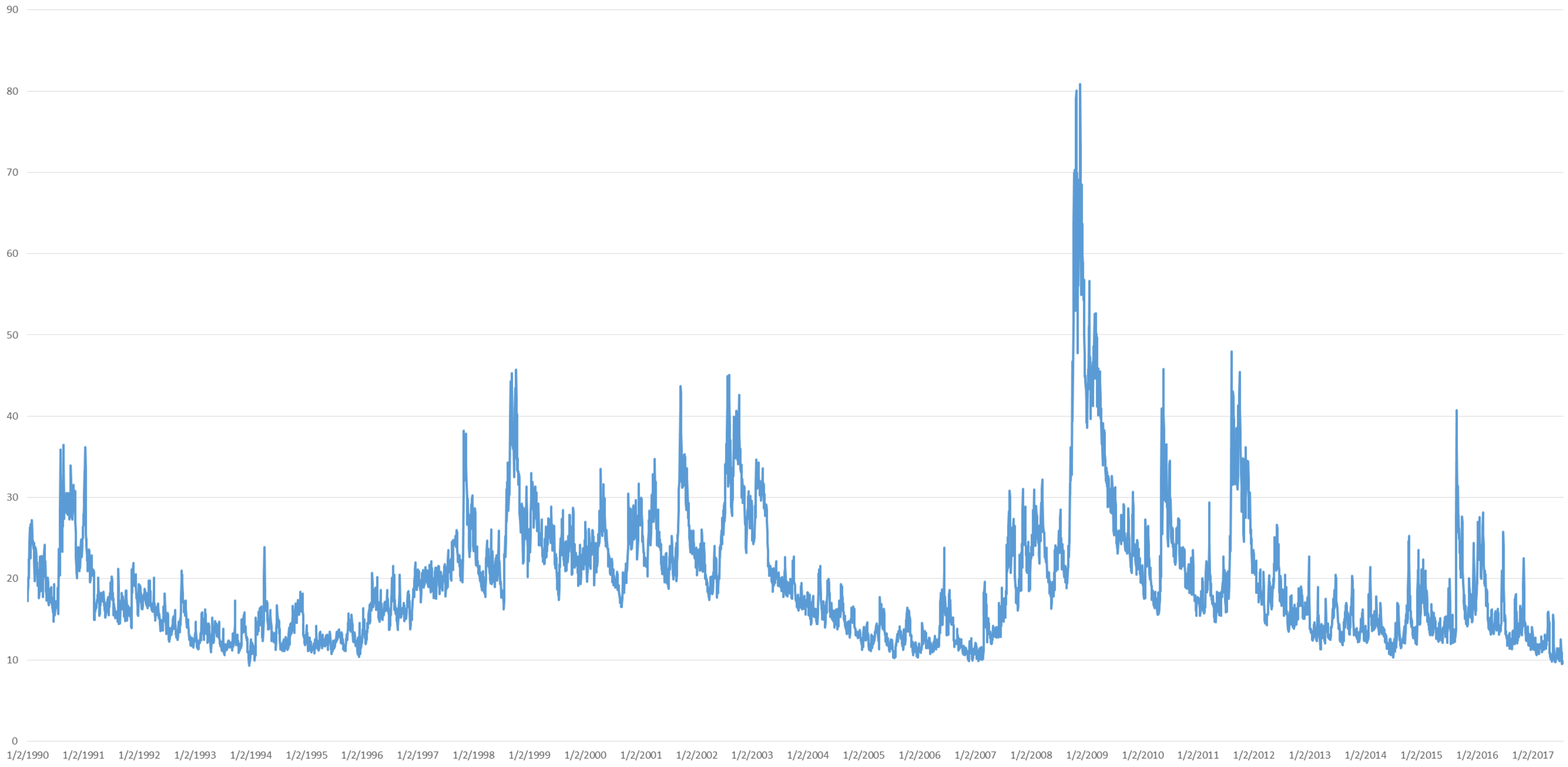
- In 2000, CBOE created a discrete version of the VSV in which the sum replaces the integral and the maturity is 30 days. Since there are no 30 day options, VIX uses first two maturities*

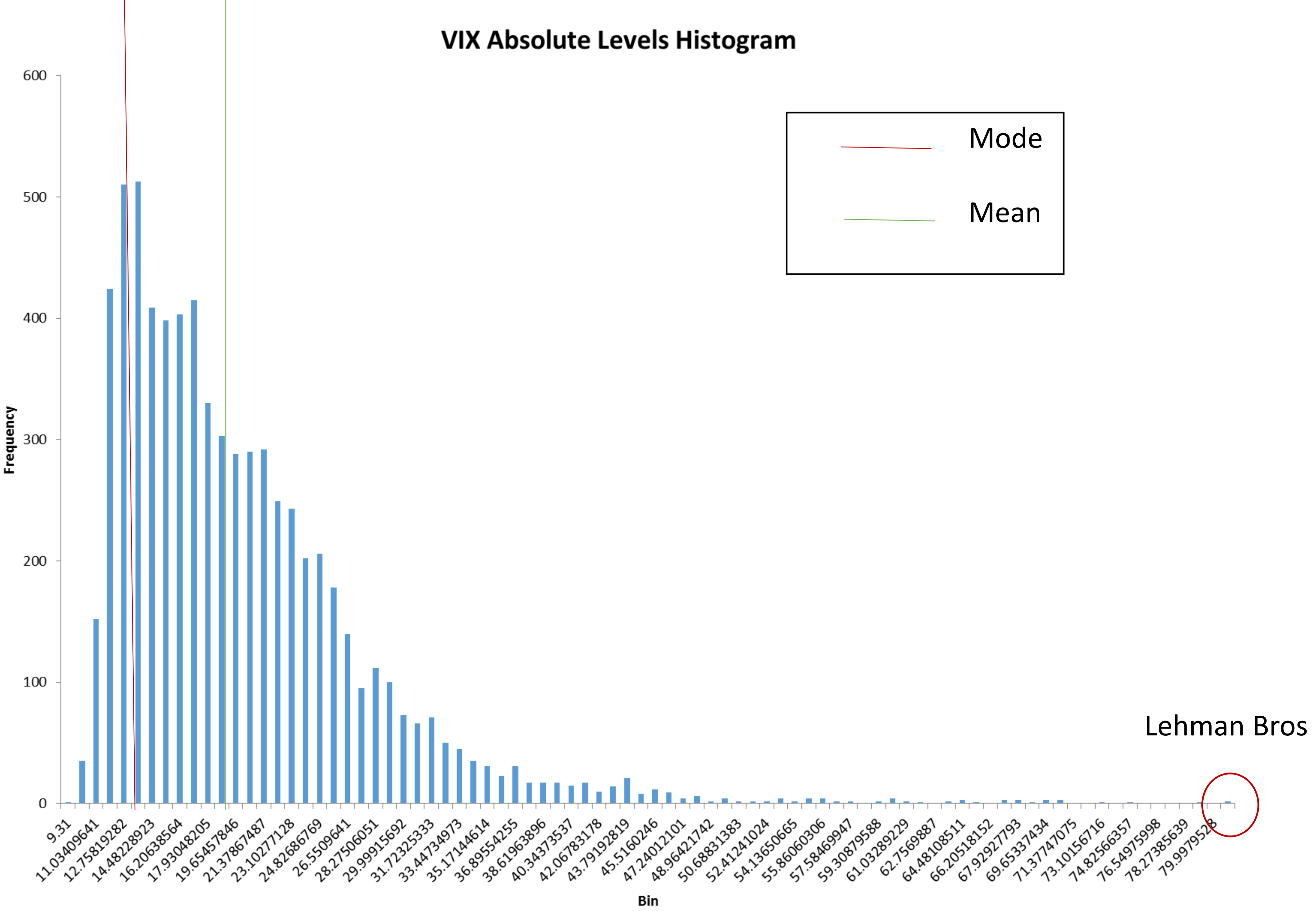
$$\boxed{VIX = \sqrt{w_1 \sum_{i=1}^{n} OTM(K_i,T_1,S)\frac{\Delta K}{K_i^2} + w_2 \sum_i OTM(K_i,T_2,S)\frac{\Delta K}{K_i^2}}}$$

* My understanding is that recently they could have added more maturities using weekly options as well.

VIX: Jan 1990 to July 2017

Is VIX mean-reverting/stationary?

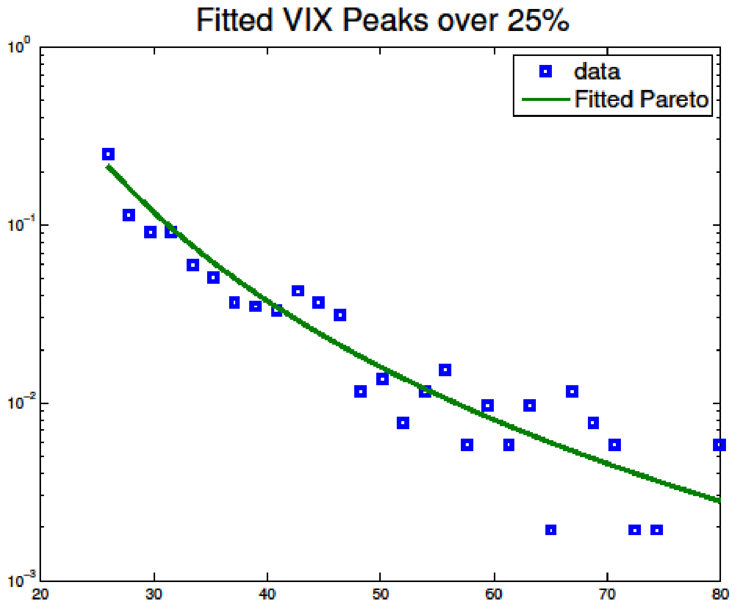




VIX Absolute Levels Histogram
Frequency
600
500
400
300
200
100
0
Mode
Mean
Lehman Bros
9.31
11.03409641
12.75819282
14.48228923
16.20638564
17.93048205
19.65457846
21.37867487
23.10277128
24.82686769
26.5509641
28.27506051
29.99915692
31.72325333
33.44734973
35.17144614
36.89554255
38.61963896
40.34373537
42.06783178
43.79192819
45.5160246
47.24012101
48.96421742
50.68831383
52.41241024
54.13650665
55.86060306
57.58469947
59.30879588
61.03289229
62.7569887
64.48108511
66.20518152
67.92927793
69.65337434
71.37747075
73.10156716
74.82566357
76.54975998
78.27385639
79.9979528
Bin

VIX Descriptive Statistics

VIX Descriptive Stats	
Mean	19.51195
Standard Error	0.094278
Median	17.63
Mode	11.57
Standard Deviation	7.855663
Sample Variance	61.71144
Kurtosis	7.699637
Skewness	2.1027
Minimum	9.31
Maximum	80.86



- Definitely heavy tails
- ``Vol risk premium theory'' implies long-dated futures prices should be above the average VIX.
- This implies that the typical futures curve should be upward sloping (contango) since mode<average

Is VIX a stationary process? Yes and no…

- Augmented Dickey-Fuller test rejects unit root if we consider data since 1990.
  MATLAB adftest(): DFstat=-3.0357; critical value CV= -1.9416; p-value=0.0031.

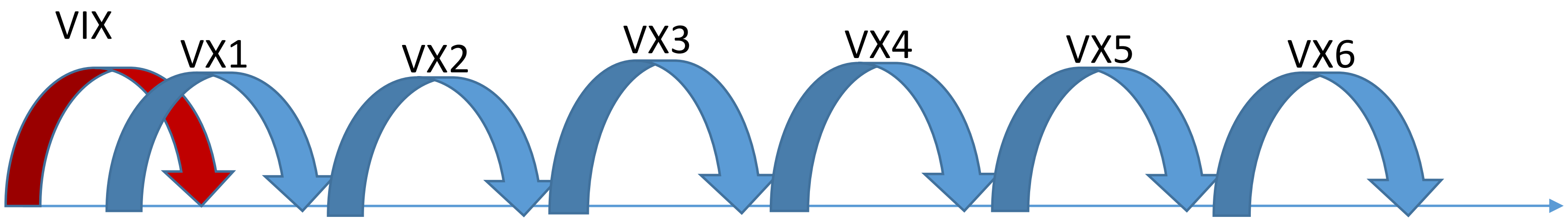
- Shorter time-windows, which don't include 2008, do not reject unit root

- Non-parametric approach (2-sample KS test) rejects unit root if 2008 is included.

- We shall assume stationarity and explore some of its consequences
  in the real-world investing

VIX Futures (symbol:VX)

- Contract notional value = VX $\times$ 1,000
- Tick size= 0.05 (USD 50 dollars)
- Settlement price = VIX $\times$ 1,000
- Monthly settlements, on Wednesday at 8AM, prior to the 3rd Friday (classical option expiration date)
- Exchange: Chicago Futures Exchange (CBOE)
- Cash-settled (obviously)



- Each VIX futures covers 30 days of volatility after the settlement date.
- Settlement dates are 1 month apart.
- Recently, weekly settlements have been added in the first two months.

VIX Central

Questions and Feedback

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VIX futures 6:30 PM
Thursday Sep 14, 2017

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VIX Futures Term Structure
Source: CBOE Delayed Quotes
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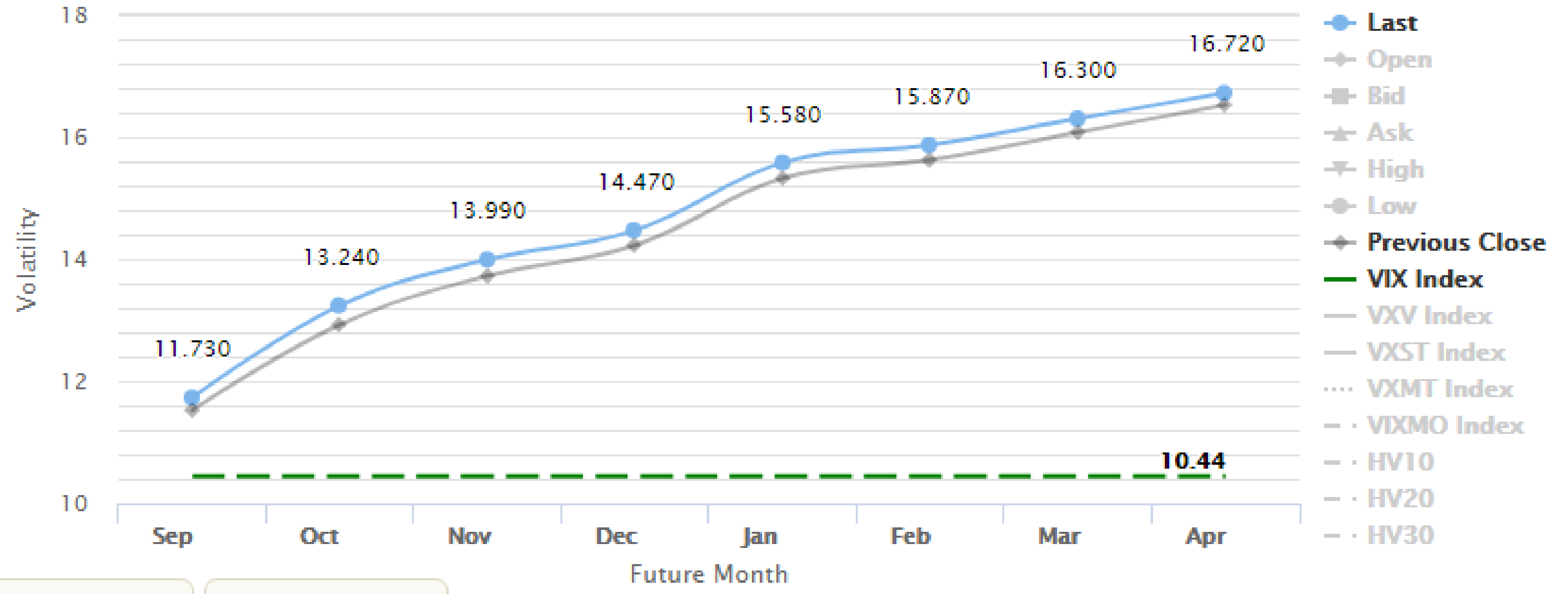

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% Contango	1	12.87%	2	5.66%	3	3.43%	4	7.67%	5	1.86%	6	2.71%	7	2.58%	8
Difference	1	1.51	2	0.75	3	0.48	4	1.11	5	0.29	6	0.43	7	0.42	8

Month 7 to 4 contango	12.65%	4.22%

Settlement dates:

Sep 20, 2017

Oct 18, 2017

Nov 17, 2017

Dec 19, 2017

Jan 16, 2018

Feb 13, 2018

Mar 20, 2018

April 17, 2018

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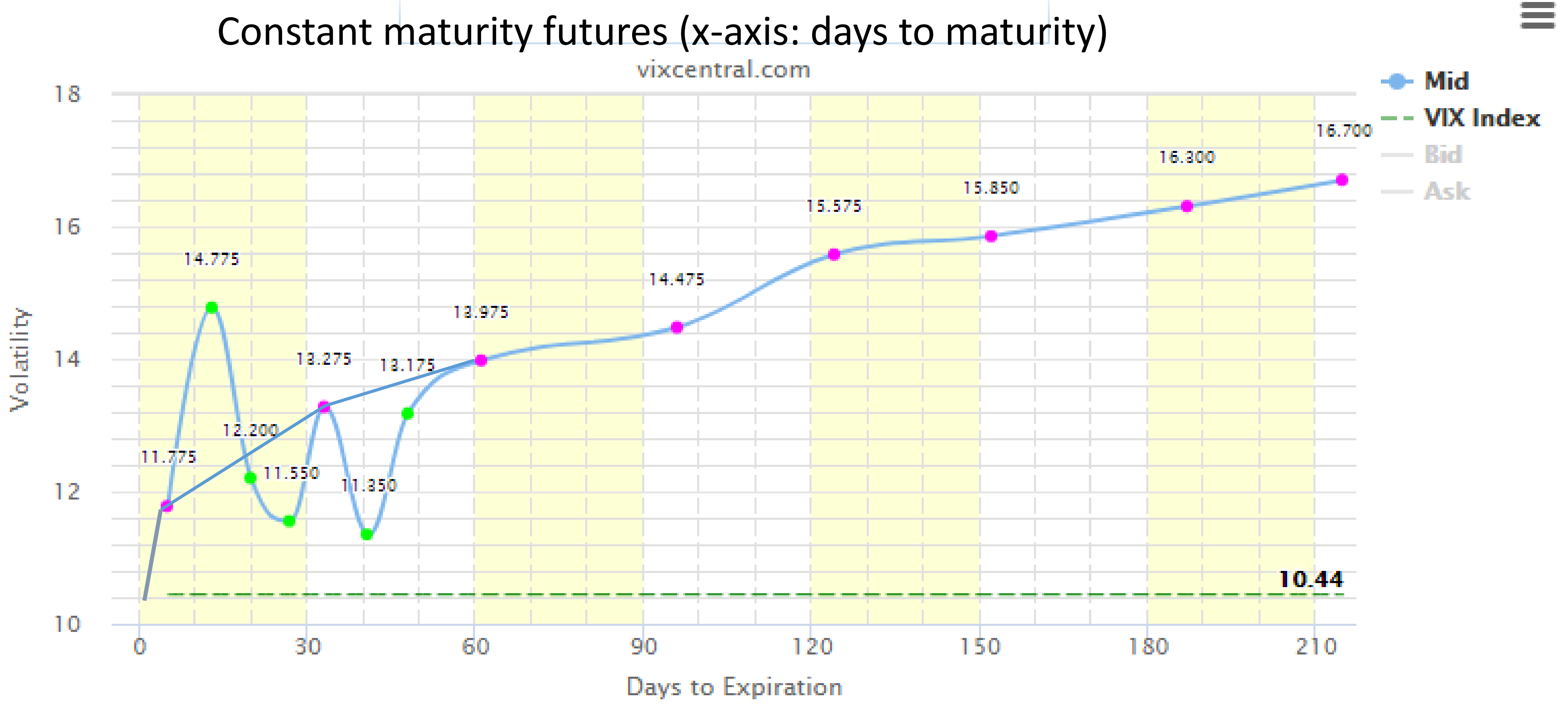

Note: Recently introduced weeklies are illiquid and should not be used to build CMF curve

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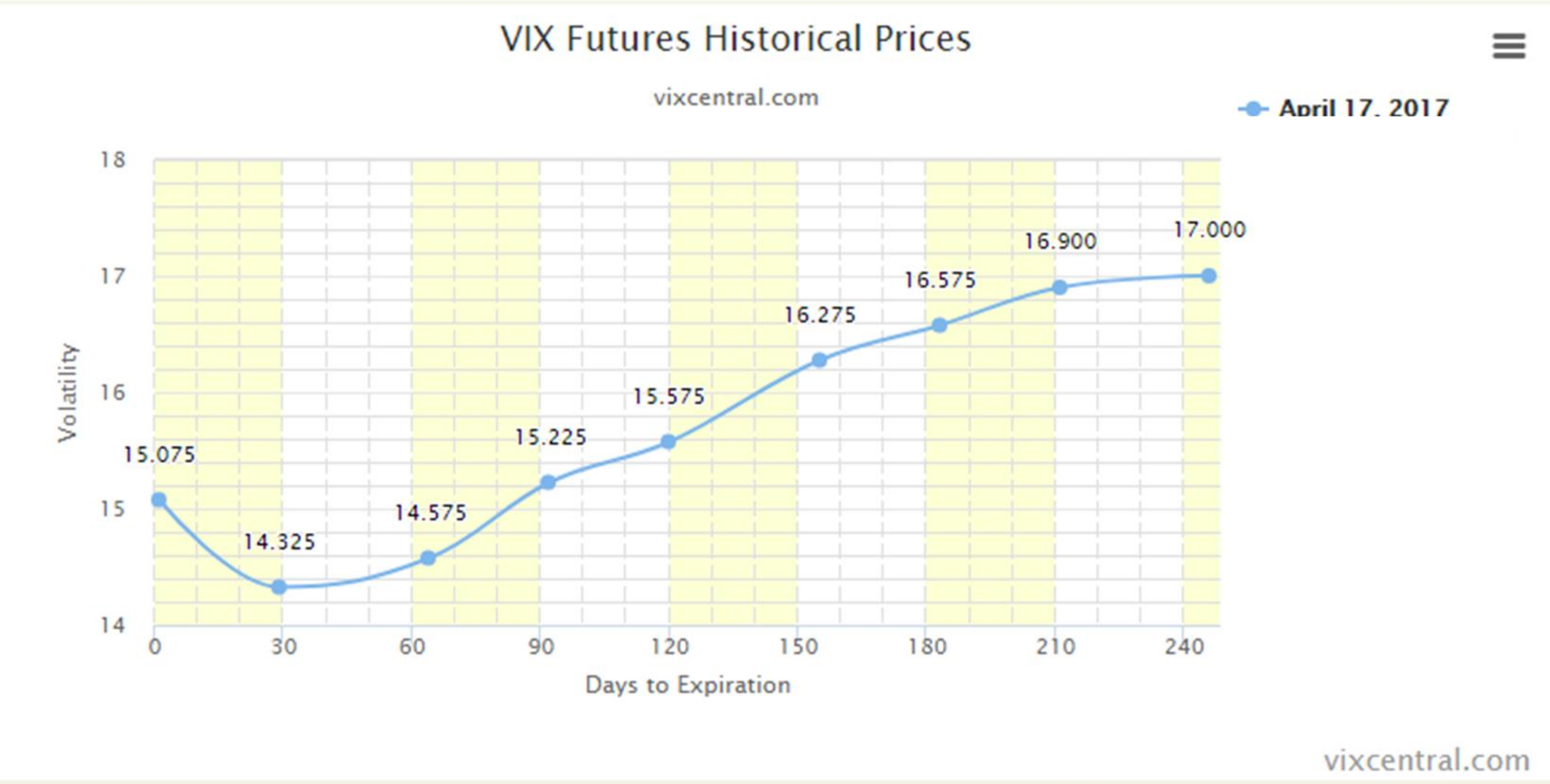
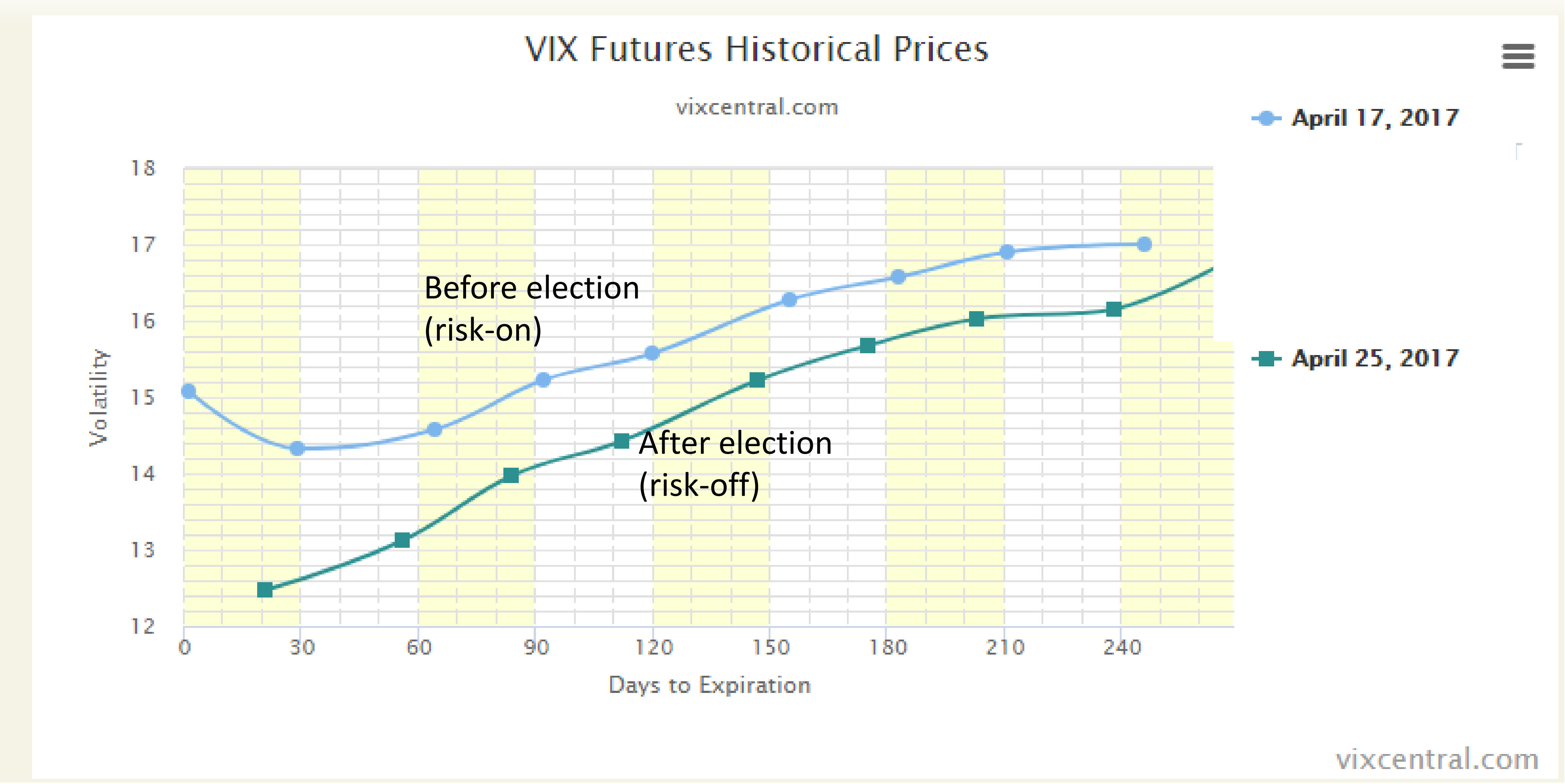
Partial Backwardation: French election, 1st round



Term-structures before & after French election



VIX futures: Lehman week, and 2 months later

A paradigm for the VIX futures cycle

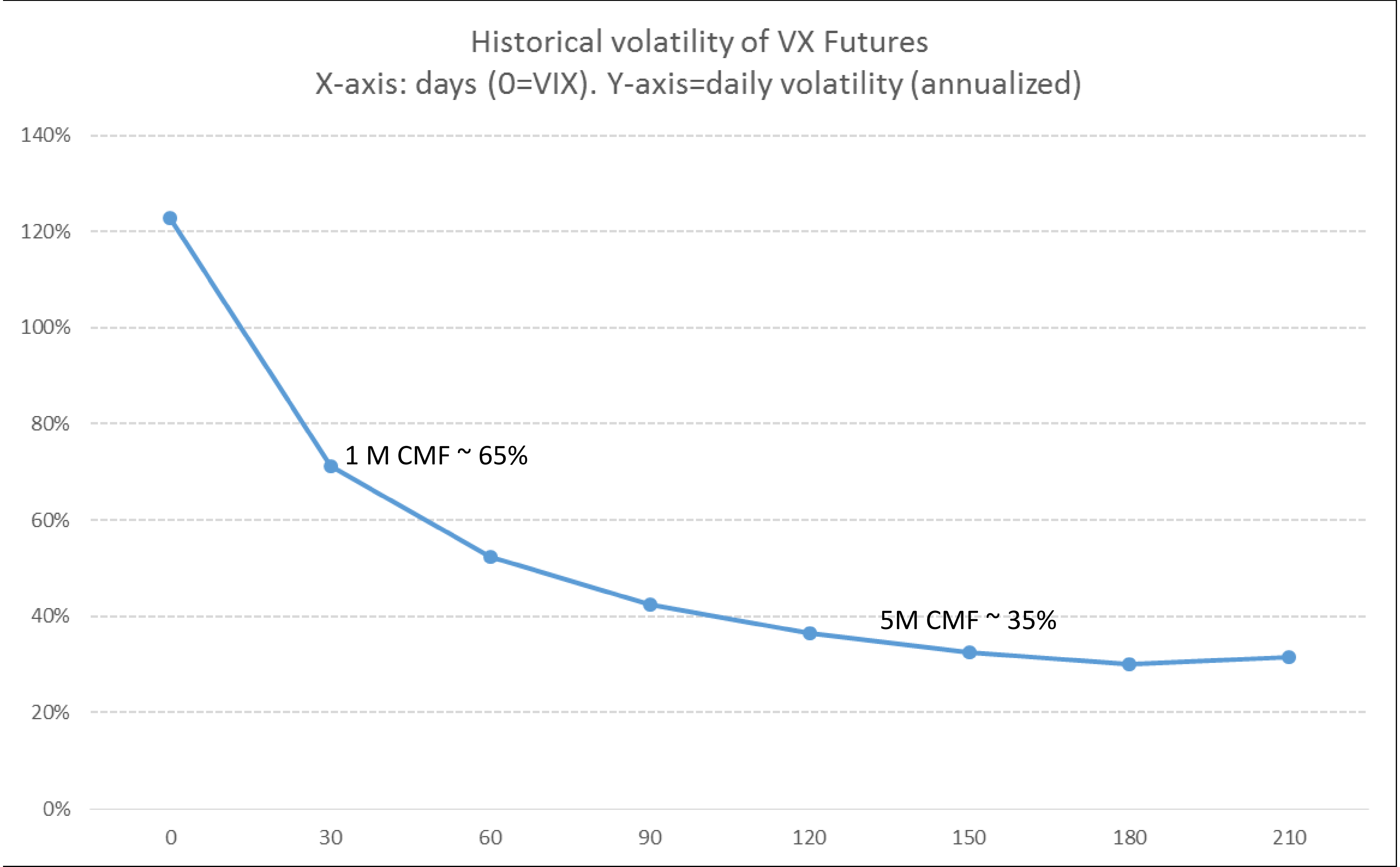
- Markets are ``quiet'', volatility is low , VIX term structure is in contango (i.e. upward sloping)
- Risk on: the possibility of market becoming more risky arises; 30-day S&P implied vols rise
- VIX spikes, CMF flattens in the front , then curls up, eventually going into backwardation
- Backwardation is usually partial (CMF decreases only for short maturities), but can be total in extreme cases (2008)
- Risk-off: uncertainty resolves itself, CMF drops and steepens
- Most likely state (contango) is restored

Statistics of VIX Futures

- Constant-maturity futures, V^{τ}, linearly interpolating quoted futures prices

$$V_t^{\tau} = \frac{\tau_{k+1} - \tau}{\tau_{k+1} - \tau_k} VX_k(t) + \frac{\tau - \tau_k}{\tau_{k+1} - \tau_k} VX_{k+1}(t)$$

VX_k(t)= kth futures price on date t, $VX_0 =$ VIX, $\tau_0 = 0$, $\tau_k =$ tenor of kth futures


Historical volatility of VX Futures
X-axis: days (0=VIX). Y-axis=daily volatility (annualized)
140%
120%
100%
80%
60%
40%
20%
0%
0
30
60
90
120
150
180
210
1 M CMF ~ 65%
5M CMF ~ 35%

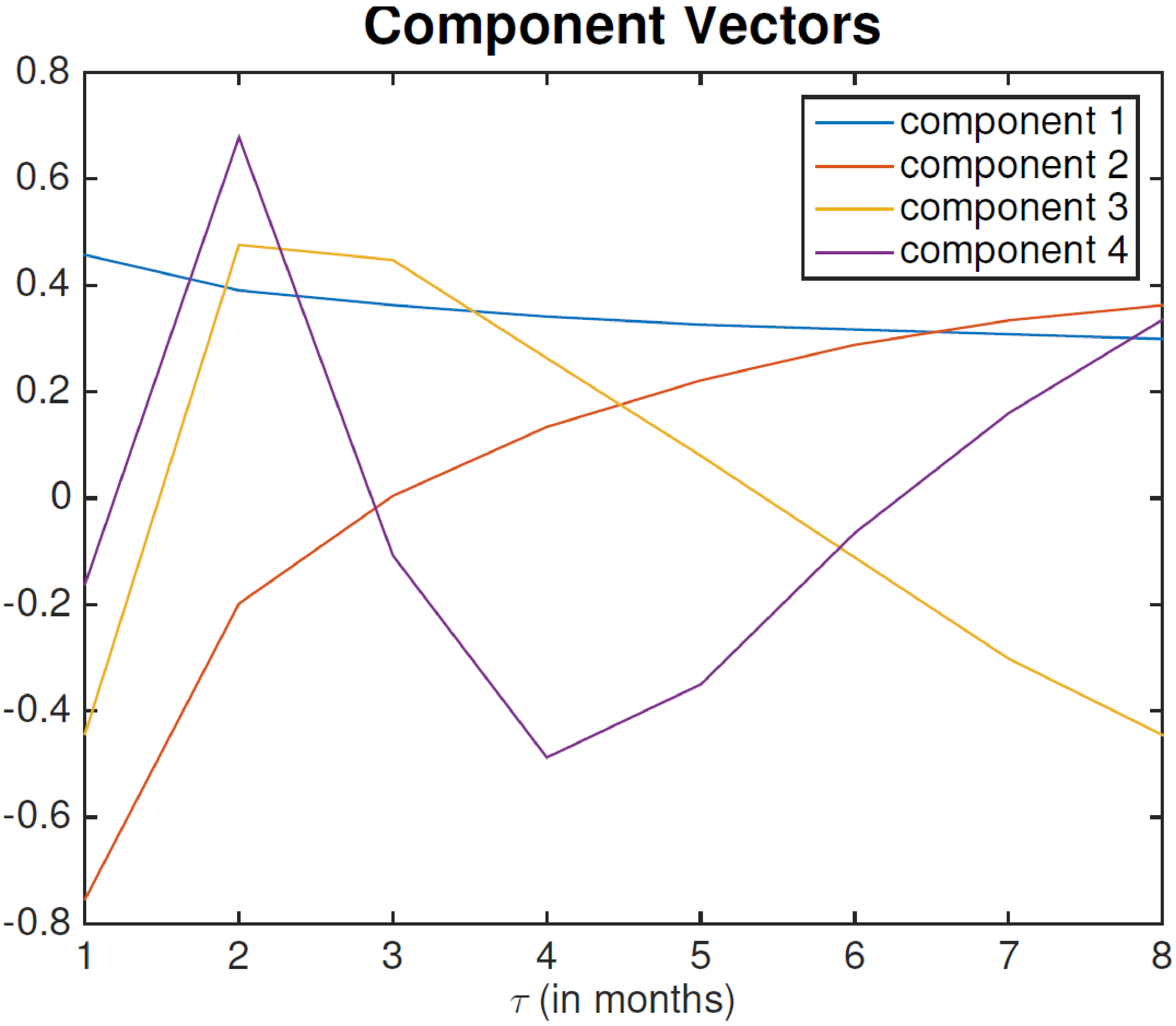
PCA: fluctuations from average position

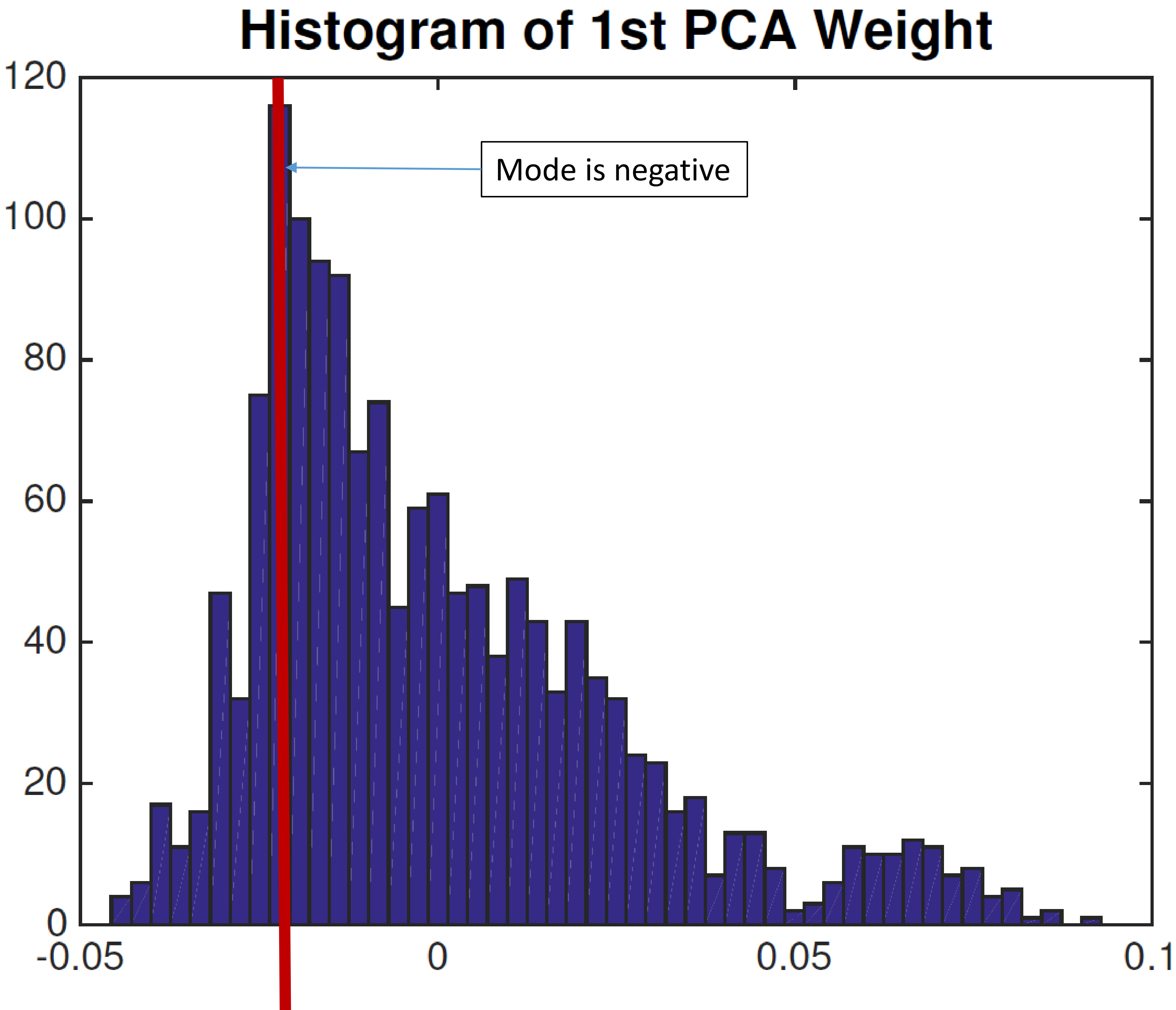
- Select standard tenors τ_k, $k = 0, 30, 60, 90, 120,150, 180, 210$
- Dates: Feb 8 2011 to Dec 15 2016

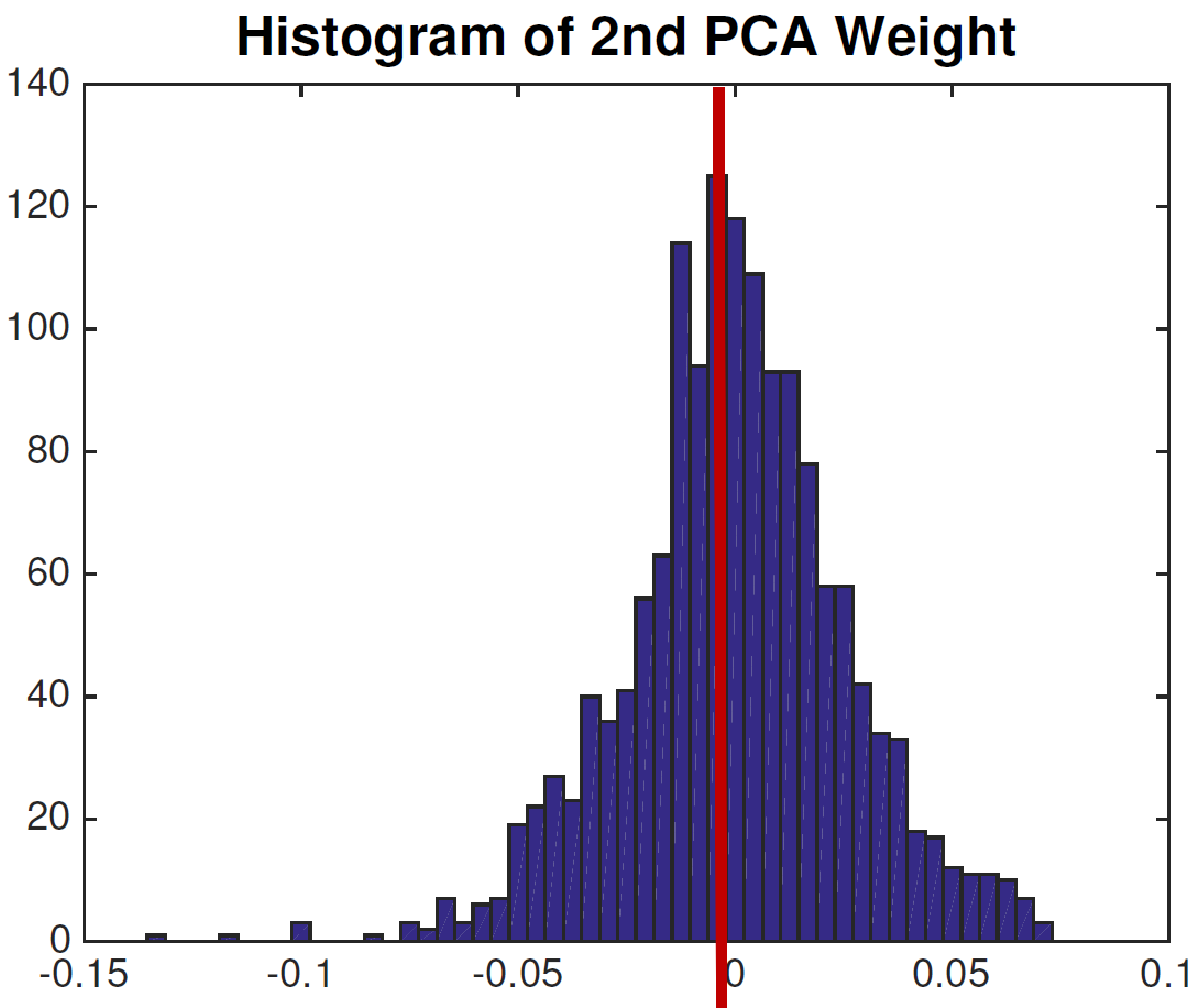
$$lnV_{t_i}^{\tau_k} = \overline{lnV^{\tau_k}} + \sum_{l=1}^{8} a_{il}\Psi_l^k$$

- Slightly different from Alexander and Korovilas (2010) who did the PCA of 1-day log-returns.

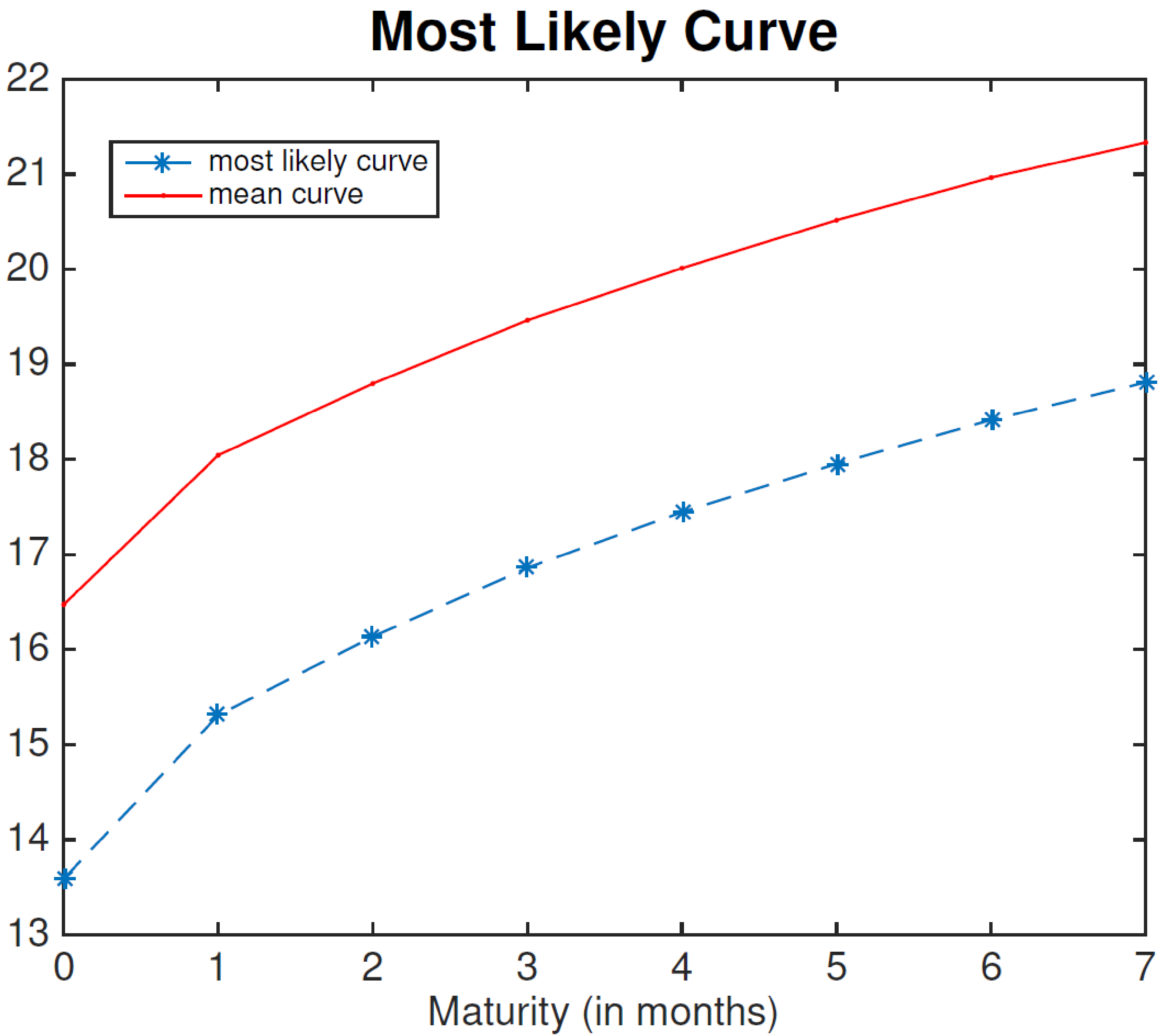
Eigenvalue	% variance expl
1	72
2	18
3	6
4	1
5 to 8	<1


Component Vectors
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1
2
3
4
5
6
7
8
τ (in months)
component 1
component 2
component 3
component 4


Histogram of 1st PCA Weight
Mode is negative
120
100
80
60
40
20
0
-0.05
0
0.05
0.1



Histogram of 2nd PCA Weight
140
120
100
80
60
40
20
0
-0.15
-0.1
-0.05
0
0.05
0.1


Most Likely Curve
most likely curve
mean curve
Maturity (in months)
22
21
20
19
18
17
16
15
14
13
0
1
2
3
4
5
6
7

ETFs/ETNs based on futures

- Use futures contracts as a proxy to the commodity itself

- Track an ``investable index'', corresponding to a rolling futures strategy

- Fund invests in a basket of futures contracts

$$\frac{dI}{I} = r\,dt + \sum_{i=1}^{N} a_i \,\frac{dF_i}{F_i}$$

a_i = fraction (%) of assets in ith future

- Normalization

$$\sum_{i=1}^{N} a_i = \beta,$$

β = leverage coefficient

Average maturity

- Assume $\beta = 1$, let b_i= fraction of **total number of contracts** invested in i[th] futures:

$$b_i = \frac{n_i}{\sum n_j} = \frac{I\, a_i}{F_i}.$$

- The average maturity θ is defined as

$$\theta = \sum_{i=1}^{N} b_i\,(T_i - t) = \sum_{i=1}^{N} b_i \tau_i$$

Example 1: VXX (maturity = 1M, long futures, daily rolling)

$$\frac{dI}{I} = rdt + \frac{b(t)dF_1 + (1-b(t))dF_2}{b(t)F_1 + (1-b(t))F_2}$$

Weights are based on 1-M CMF , no leverage

$$b(t) = \frac{T_2 - t - \theta}{T_2 - T_1}$$

$$\theta = 1 \text{ month} = 30/360$$

Notice that since $V_t^\theta = b(t)F_1 + (1-b(t))F_2$

we have $dV_t^\theta = b(t)dF_1 + (1-b(t))dF_2 + b'(t)F_1 - b'(t)F_2$

Hence

$$\frac{dV_t^\theta}{V_t^\theta} = \frac{b(t)dF_1 + (1-b(t))dF_2}{b(t)F_1 + (1-b(t))F_2} + \frac{F_2 - F_1}{b(t)F_1 + (1-b(t))F_2}\frac{dt}{T_2 - T_1}$$

Dynamic link between Index and CMF equations (long 1M CMF, daily rolling)

$$\frac{dI}{I} = rdt + \frac{b(t)dF_1 + (1 - b(t))dF_2}{b(t)F_1 + (1 - b(t))F_2}$$

$$= r\,dt + \frac{dV_t^\theta}{V_t^\theta} - \frac{F_2 - F_1}{b(t)F_1 + (1 - b(t))F_2}\frac{dt}{T_2 - T_1}$$

$$\boxed{\frac{dI}{I} = r\,dt + \frac{dV_t^\theta}{V_t^\theta} - \left.\frac{\partial \ln V_t^\tau}{\partial\,\tau}\right]_{\tau=\theta} dt}$$

Slope of the CMF is the relative drift between index and CMF

Example 2 : XIV, Short 1-M rolling futures

This is a fund that follows a DAILY rolling strategy, sells futures, targets 1-month maturity

$$\frac{dJ}{J} = r\,dt - \frac{dV_t^{\theta}}{V_t^{\theta}} + \left.\frac{\partial \ln V_t^{\tau}}{\partial\,\tau}\right]_{\tau=\theta} dt$$

$$\theta = 1 \text{ month} = 30/360$$

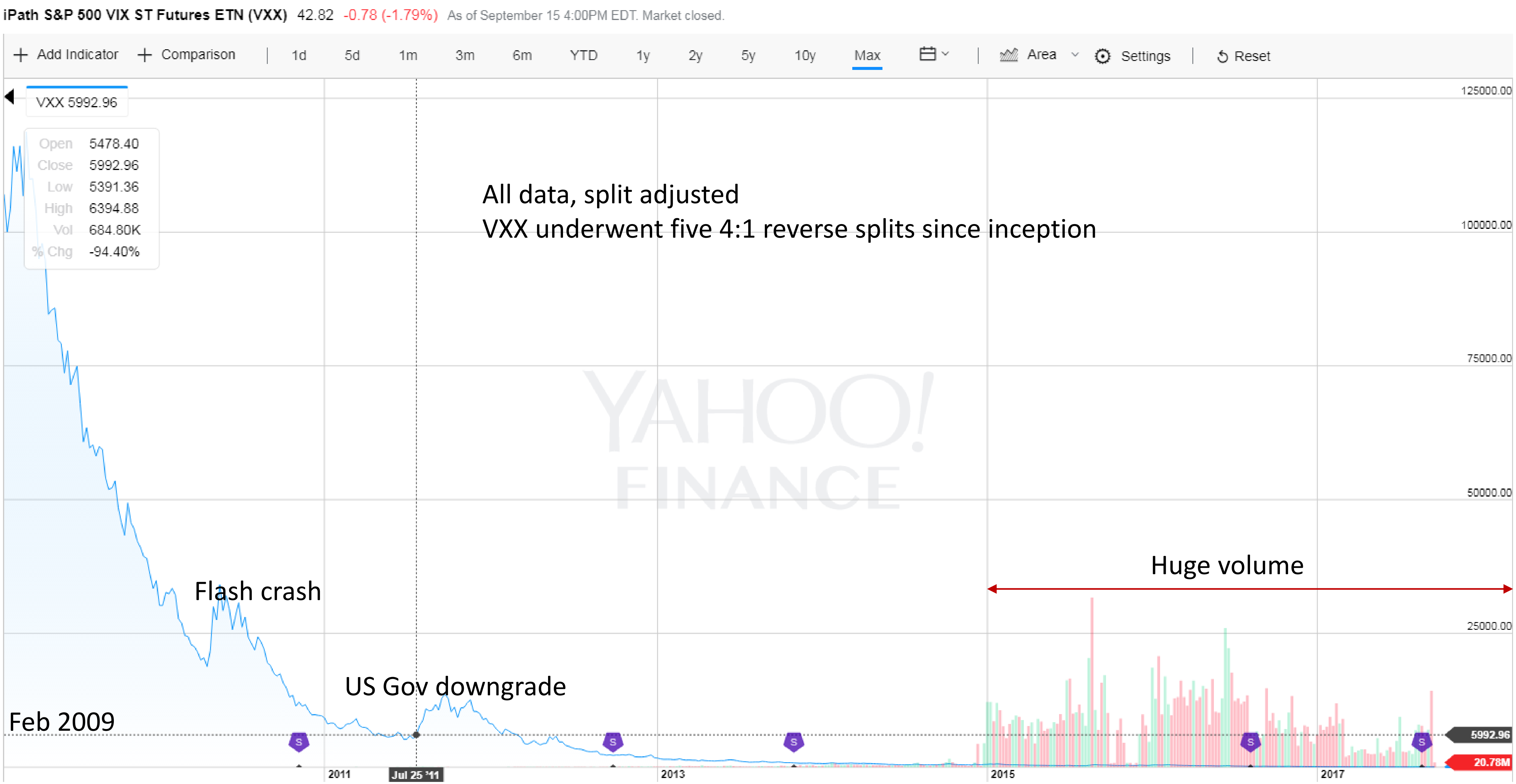
Stationarity/ergodicity of CMF and consequences

Integrating the *I*-equation for VXX and the corresponding *J*-equation for XIV (inverse):

$$VXX_0 - e^{-r\,t}\,VXX_t = VXX_0\left[1 - \frac{V_t^{\tau}}{V_0^{\tau}}\,exp\left(-\int_0^t \frac{\partial \ln V_s^{\theta}\,ds}{\partial \tau}\right)\right]$$

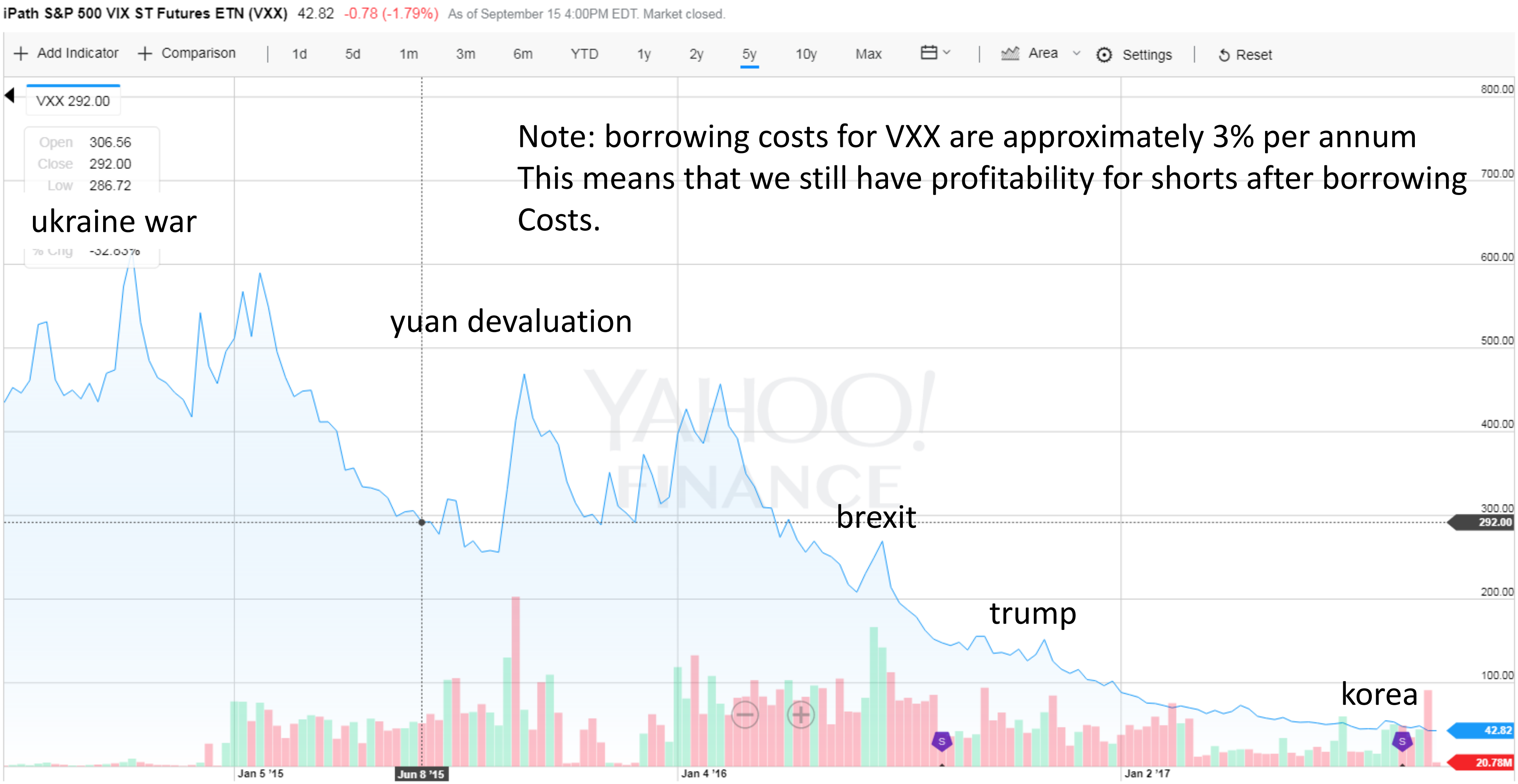
$$e^{-r\,t}\,XIV_t - XIV_0 = XIV_0\left[\frac{V_0^{\tau}}{V_t^{\tau}}\,exp\left(\int_0^t \frac{\partial \ln V_s^{\theta}\,ds}{\partial \tau}\right) - 1\right]$$

Proposition: If VIX is stationary and ergodic, and $E\left(\frac{\partial \ln V_s^{\theta}}{\partial \tau}\right) > 0,$ static buy-and-hold XIV or short-and-hold VXX produce sure profits in the long run, with probability 1.



iPath S&P 500 VIX ST Futures ETN (VXX) 42.82 -0.78 (-1.79%) As of September 15 4:00PM EDT. Market closed.
Add Indicator
Comparison
1d
5d
1m
3m
6m
YTD
1y
2y
5y
10y
Max
Area
Settings
Reset
VXX 5992.96
Open 5478.40
Close 5992.96
Low 5391.36
High 6394.88
Vol 684.80K
% Chg -94.40%
All data, split adjusted
VXX underwent five 4:1 reverse splits since inception
125000.00
100000.00
75000.00
50000.00
25000.00
Flash crash
US Gov downgrade
Feb 2009
Huge volume
5992.96
20.78M
2011
Jul 25 '11
2013
2015
2017

Taking a closer look, last 2 1/2 years



Last 6 months

VelocityShares Daily Inverse VIX ST ETN (XIV) 89.61 +1.69 (1.92%) As of September 15 4:00PM EDT. Market closed.
Add Indicator
Comparison
1d
5d
1m
3m
6m
YTD
1y
2y
5y
10y
Max
Area
Settings
Reset
XIV 13.89
Open 14.31
Close 13.89
Low 13.33
High 14.43
Vol 41.11M
% Chg 25.80%
korea
le pen
china
trump
brexit
100.00
90.00
89.61
80.00
70.00
60.00
50.00
40.00
30.00
20.00
13.89
10.00
5.00M
2011
Aug 27 '12
2013
2015
2017

Modeling CMF curve dynamics

- VIX ETNs are exposed to (i) volatility of VIX (ii) slope of the CMF curve
- To quantify the profitability of described short VXX/long XIV strategies, we propose a stochastic model and estimate it.
- 1-factor model is not sufficient to capture observed ``partial backwardation'' and ``bursts'' of volatility
- Parsimony suggests a 2-factor model
- We build-in mean-reversion to investigate the stationarity assumptions
- Sacrifice other ``stylized facts'' (fancy vol-of-vol) to obtain analytically tractable formulas

`Classic’ log-normal 2-factor model for VIX

$$VIX_t = exp(X_{1\,t} + X_{2\,t})$$

$$dX_1 = \sigma_1 dW_1 + k_1(\mu_1 - X_1)dt$$

$$dX_2 = \sigma_2 dW_2 + k_2(\mu_2 - X_2)dt$$

$$dW_1\, dW_2 = \rho\, dt$$

X_1 = factor driving mostly VIX or short-term futures fluctuations (slow)

X_2 = factor driving mostly CMF slope fluctuations (fast)

These factors should be positively correlated.

Constant Maturity Futures

$$V^{\tau} = E^{Q}\{VIX_{\tau}\} = E^{Q}\{exp(X_{1\,\tau} + X_{2\,\tau})\}$$

Ensuring no-arbitrage between Futures, Q = ``pricing measure'' with MPR

$$V^{\tau} = V^{\infty}\, exp\left[e^{-\bar{k}_1\tau}(X_1 - \bar{\mu}_1) + e^{-\bar{k}_2\tau}(X_2 - \bar{\mu}_2) - \frac{1}{2}\sum_{ji=1}^{2}\frac{e^{-\bar{k}_i\tau}e^{\overline{-k}_j\tau}}{\bar{k}_i+\bar{k}_j}\sigma_i\sigma_j\rho_{ij}\right]$$

Here, the `overline parameters' correspond to assuming a linear market price of risk, which makes the risk factors X distributed like OU processes under Q with renormalized parameters

Estimating the model consists in finding $k_1, \mu_1, k_2, \mu_2, \bar{k}_1, \bar{\mu}_1, \bar{k}_2, \bar{\mu}_2, \sigma_1, \sigma_2, \rho, V^{\infty}$ using historical data

Stochastic differential equations for ETNs (e.g. VXX)

$$\frac{dI}{I} = r\,dt + \frac{dV_t^\theta}{V_t^\theta} - \left.\frac{\partial \ln V_t^\tau}{\partial\,\tau}\right]_{\tau=\theta} dt$$

Substituting closed-form solution in the ETN index equation we get:

$$\frac{dI}{I} = r\,dt + \sum_{i=1}^{2} e^{-\bar{k}_i\theta}\sigma_i dW_i + \sum_{i=1}^{2} e^{-\bar{k}_i\theta}\left[\bar{k}_i(\mu_i - \bar{\mu}_i) + \left(k_i - \bar{k}_i\right)(X_i - \mu_i)\right] dt$$

$$\text{Equilibrium local drift} = \sum_{i=1}^{2} e^{-\bar{k}_i\theta}\left[\bar{k}_i(\mu_i - \bar{\mu}_i)\right] + r$$

$$\text{Local variance} = \sum_{ji=1}^{2} e^{-\bar{k}_i\tau} e^{\overline{-k}_j\tau}\, \sigma_i\sigma_j\rho_{ij}$$

Estimating the model, 2011-2016

- Kalman filtering approach

Estimated Θ
Input data: 2/2011 to 12/2016, with VIX and CMFs 1m to 7m.

$\bar{\mu}_1$	3.8103	μ_1	3.2957
$\bar{\mu}_2$	-0.7212	μ_2	-0.7588
$\bar{\kappa}_1$	1.1933	κ_1	0.4065
$\bar{\kappa}_2$	10.8757	κ_2	13.1019
σ_1	0.6776		
σ_2	0.8577		
ρ	0.4462		

Estimated Θ
Input data: 2/2011 to 12/2016, with VIX, 3m and 6m CMFs.

$\bar{\mu}_1$	3.8094	μ_1	3.2524
$\bar{\mu}_2$	-0.7100	μ_2	-0.7155
$\bar{\kappa}_1$	1.0215	κ_1	0.3219
$\bar{\kappa}_2$	10.8739	κ_2	13.0799
σ_1	0.5931		
σ_2	0.9066		
ρ	0.4266		

Estimating the model, 2007 to 2016 (contains 2008)

Estimated Θ
Input data: 7/2007 to 7/2016, with VIX and CMFs 1m to 6m.

$\bar{\mu}_1$	-6.6216	μ_1	-7.1367
$\bar{\mu}_2$	9.7372	μ_2	9.7608
$\bar{\kappa}_1$	0.6543	κ_1	0.2010
$\bar{\kappa}_2$	5.9052	κ_2	5.9389
σ_1	0.5525		
σ_2	0.9802		
ρ	0.6015		

Estimated Θ
Input data: 7/2007 to 7/2016, with VIX, 1m and 6m CMFs.

$\bar{\mu}_1$	2.4581	μ_1	1.8685
$\bar{\mu}_2$	0.8002	μ_2	0.7555
$\bar{\kappa}_1$	0.5505	κ_1	0.1081
$\bar{\kappa}_2$	10.0013	κ_2	12.3600
σ_1	0.4294		
σ_2	0.7998		
ρ	0.5073		

Results of the Numerical Estimation:

Model's prediction of profitability for short VXX/long XIV, in equilibrium

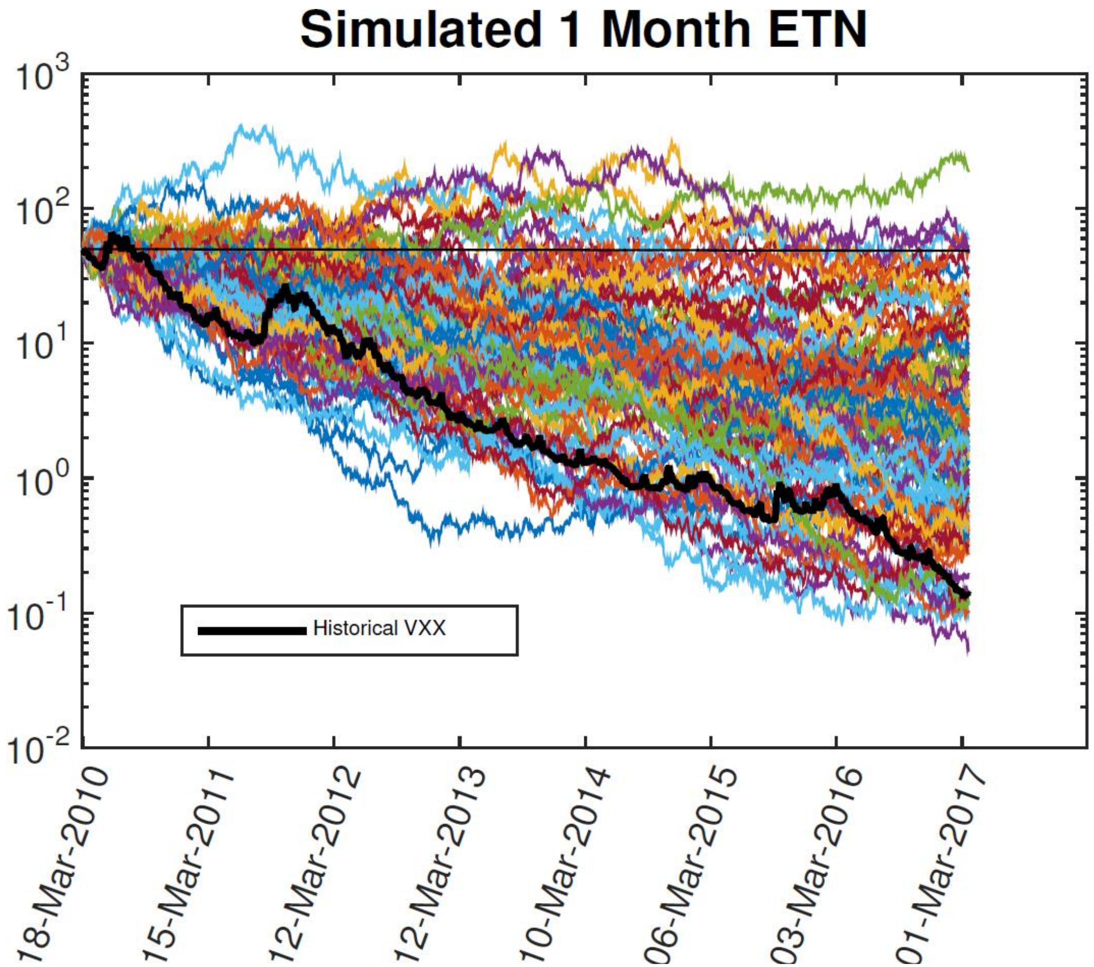
	Jul 07 to Jul 16 VIX, CMF 1M to 6M	Jul 07 to Jul 16 VIX, 1M, 6M	Feb 11 to Dec 16 VIX, CMF 1M to 7M	Feb 11 to Jul 16 VIX, 3M, 6M
Excess Return	0.30	0.32	0.56	0.53
Volatility	1.00	0.65	0.82	0.77
Sharpe ratio (short trade)	0.29	0.50	0.68	0.68

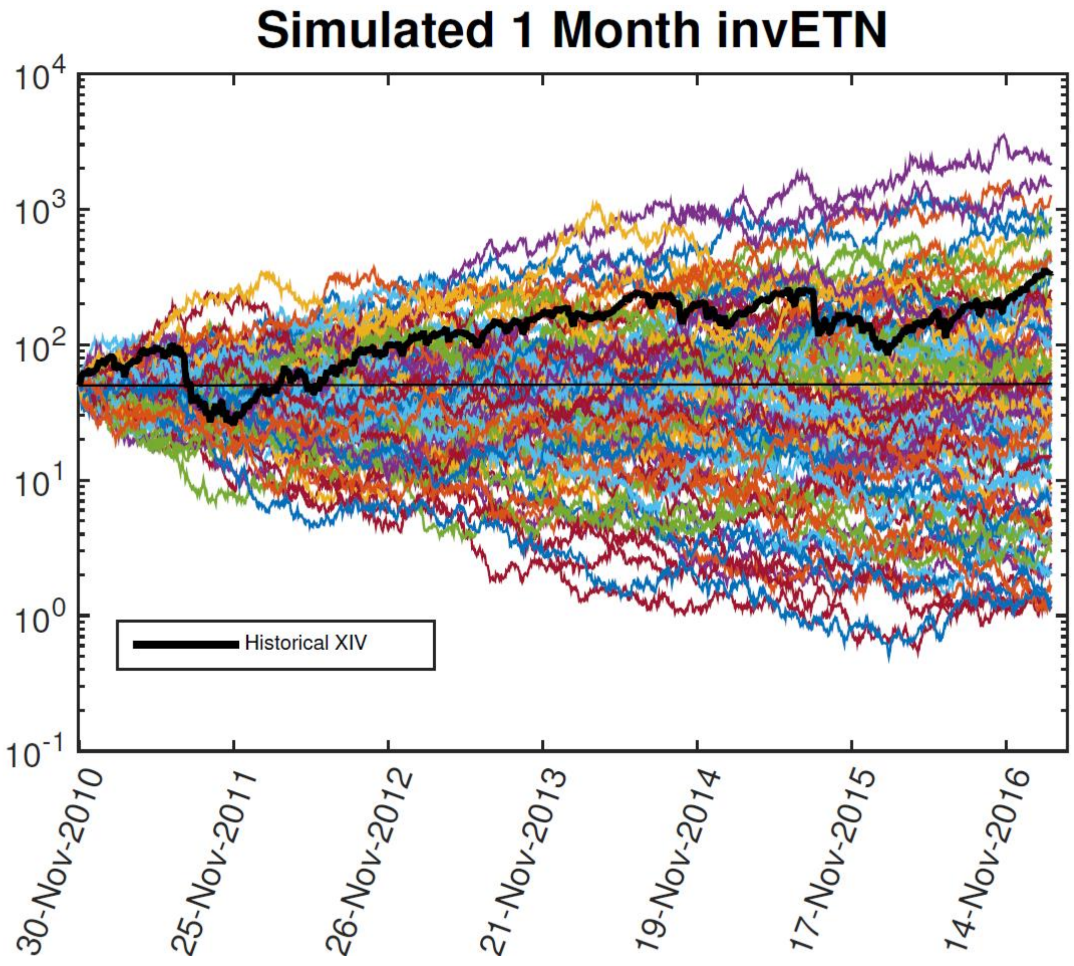
Notes :

(1) For shorting VXX one should reduce the ``excess return'' by the average borrowing cost which is 3%.
It is therefore better to be long XIV (note however that XIV is less liquid, but trading volumes in XIV are increasing.

(2) Realized Sharpe ratios are higher. For instance the Sharpe ratio for Short VXX (with 3% borrow) from Feb 11 To May 2017 is 0.90. This can be explained by low realized volatility in VIX and the fact that the model predicts significant fluctuations in P/L over finite time-windows.

Variability of rolling futures strategies predicted by model





Thank you!