All For One ... One For All?:

A Principal Component Analysis of

Latin American Brady Bond Debt

from 1994 to 2000

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Abstract

We use Principal Component Analysis (PCA) to study the Brady Bond Debt of the four primary Latin American sovereign issuers: Argentina, Brazil, Mexico, and Venezuela. Our dataset covers a period of $5\frac{1}{2}$ years starting in July 1994 and consists of daily sovereign ("stripped") yield levels for the par and discount debt securities of each country. We examine the behavior of the characteristic roots and eigenvectors of the empirical covariance matrices computed sequentially over different periods. We show that, by and large, there exist two statistically significant components, or factors, which explain up to 90% of the realized variance. The eigenvector with largest eigenvalue corresponds to the variance attributable to "regional" ("Latin") risk. The second component strongly suggests the existence of a volatility risk factor associated to Venezuelan debt in relation to the rest of the region. A time-dependent factor analyis reveals

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that the importance of the variance explained by the factor changes over time and that this variation can be interpreted to some extent in terms of market events. In particular, we investigate the relation between the evolution of the PCA factors with the market dislocations that ocurred during the observation period, including the so-called Tequila effect, Asian flu, Ruble devaluation, and Real devaluation.

1 Introduction

During the 1970's western financial institutions lent billions of dollars to Latin American developing countries. The primary debtors, Argentina, Brazil, Mexico, and Venezuela, began to default on repayment of these obligations in the early to mid 1980's. The United States Treasury Department, under then Secretary Nicholas Brady, realized the negative consequences that massive defaults would have on the banking system and on economic development in the region. Steps to restructure the outstanding Latin American debt began in the late 1980's and early 1990's, with financial institutions forgiving portions of the outstanding loans in exchange for new, long dated securities collateralized by zero-coupon US Treasury Strips. The debt instruments which arose from this restructuring are commonly known as Brady Bonds. Although many variations of Latin American Bradys exist, including some that are backed by other types of collateral, the primary debt structure that exists today is in the form of Par and Discount Bonds.[1]

The goal of this article is to analyze the market volatility of Latin American Brady debt using historical prices and market spreads. A fundamental issue debated by investors and financial analysts – particularly during market dislocations – is the extent to which these countries are correlated, or, equivalently, the extent to which the capital markets can 'distinguish', in terms of credit risk and liquidity risk, between the different countries in the region. Does the value of the debt "move together" under market shocks? If one were to answer this question by looking at the sovereign

[1] Structurally, Par and Discount Brady Bonds differ significantly in two respects: principal structure and coupon type. While both are registered as 30-Year maturities and have rolling interest guarantees from 12 to 18 months until maturity, the former are issued at par with a fixed-rate, step-up coupon and a single payment of principal at maturity. The latter, issued at a discount with a semi-annual LIBOR floating rate coupon, have an amortizing principal schedule.

yield data for Brady Bonds in Figure 1 below, the response would be a resounding yes.

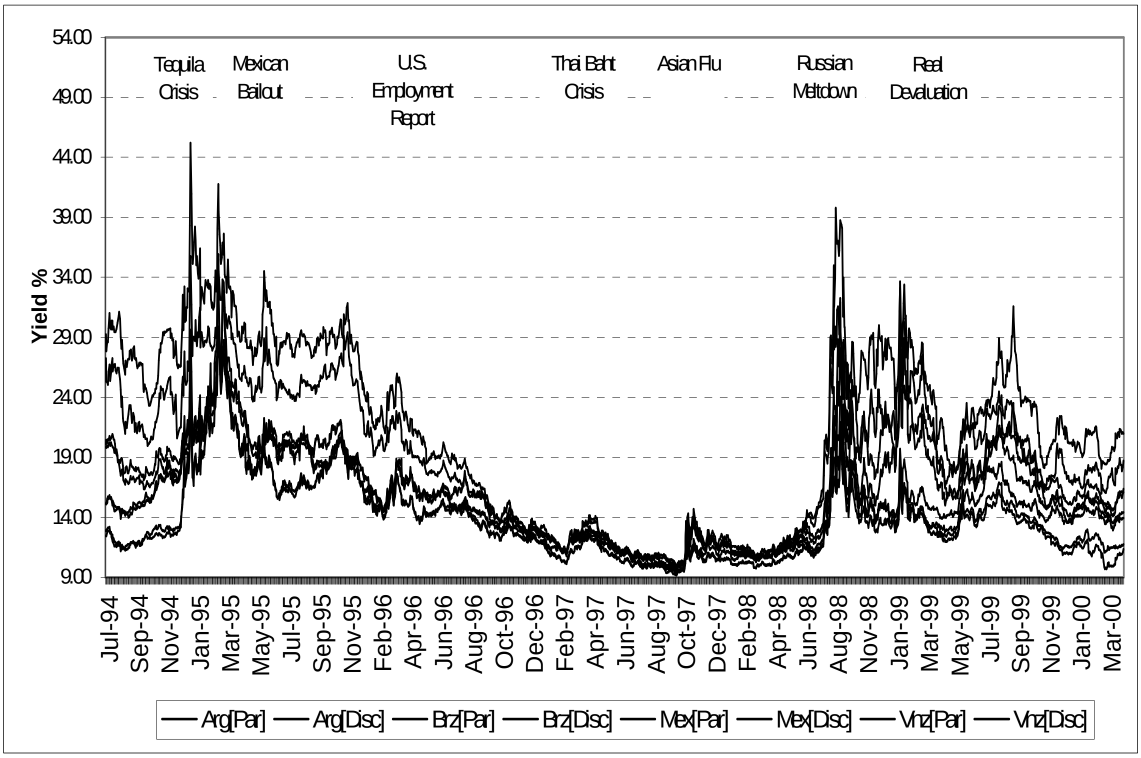

Figure 1: Latin American Brady Bond Sovereign Yields [19Jul94 to 14Apr00]

In order to provide a more precise and detailed answer, we perform a Principal Component Analysis (PCA) (Fluery, 1988) on the empirical covariance matrix constructed from daily changes in sovereign yields of Par and Discount bonds of the four countries. Mathematically speaking, the PCA provides two basic pieces of information: the *characteristic roots* (eigenvalues) and *characteristic vectors* (eigenvectors). The characteristic vectors form an orthogonal basis which is used to interpret the relationships between variables (price and yield changes, in the present case). The characteristic roots measure the contribution of each component to the total variance.[2] The overall picture offered by the PCA analysis is a decomposition of the Brady market risk into uncorrelated "shocks", or

[2] We also refer to the eigenvectors as *components*.

volatility factors. As is often the case, Principal Component Analysis is also useful to "reduce the dimension", i.e. to focus on a few important factors that describe the main sources of volatility for the market and on their economic interpretation.

2 Preliminaries

Since its inception, the Brady market has oscillated between periods of low volatility/high liquidity to periods of lengthy dislocations characterized by high volatility/low liquidity. There were also "transitional" periods between these extremes. From the econometric standpoint, the challenge is to create a framework that can capture the dynamic, non-stationary, relationships between different countries and between the bonds in each country (Pars versus Discounts).

The PCA analysis is based on the eigenvalue/eigenvector decomposition of a covariance matrix constructed from the data. There are (at least) three choices that affect the empirical covariance: the time horizon, the market valuation measure, and the valuation basis. *Ex ante*, these choices can have an impact on the results, i.e., the number of explanatory factors, the structure of the characteristic vectors and the magnitudes of the eigenvalues. Since the PCA cannot be determined in a unique fashion, we shall take into consideration different investment horizons, duration of the dislocation events, and our own market experience to interpret the data.

In particular, the choice of how frequently data measurements are made and the length of the observation period will have a significant impact on the results derived from PCA. Our objective is to balance between capturing enough detail, on the one hand, and limiting the amount of statistical noise on the other (Laloux, Bouchaud and Potters, 1999). Limited to $5\frac{1}{2}$ years of data, results derived on the basis of monthly or weekly observations proved to be dominated by noise and to lack sufficient structure. We therefore measured yield changes on a daily basis. After experimenting with several different trading window sizes ranging from 45 to 180 days, we chose to utilize a 120-day trading period window as the observation period for each of the covariance matrices.[3] The

[3] The time intervals are informally known as "windows". They reflect the number of active trading days, not

comparison of the results derived from the twelve independent covariance matrices that span the entire observation period forms the cornerstone of our research. By decomposing the market in this manner we are able to explicitly measure and interpret the impact of different economic events that have affected the Brady Bond market over the past five years.

As far as the choice of valuation basis, we consider absolute daily sovereign yield changes for the Par and Discount Brady Bonds of Argentina, Brazil, Mexico, and Venezuela (8 variables) taken over a $5\frac{1}{2}$-year period beginning in July 1994. [4] Stripped or sovereign yields represent the market's perception of a debtor country's ability to meet the non-collateralized, or non-guaranteed, portion of their debt obligations. In the context of credit option theory, sovereign yields measure the implied probability of default and reflect the market's perception of the liquidity of the debt. We verified that the results obtained for the first two components of the covariance matrix are essentially independent of the choice of valuation basis. In other words, the structure of the covariance is analogous if we consider the time-series of relative (percentage) changes in yields, cash-flow yields or prices.

Based on this "dynamic PCA" approach, we shall attempt to address the following questions: What are the main risk-components viewed by the market on the basis of the covariance of yields? Can we see a difference in the PCA structure when comparing exogenous events (e.g. the Asian flu and the Ruble crisis of 1998) with endogenous events (Mexican Bailout of 1995 and Brazilian Real devaluation of 1999)? How closely correlated is the long-term and short-term behavior of the market? What impact does the price of crude oil have, if any, on the Brady Bond debt of Mexico and Venezuela due to the attachment of Value Recovery Rights or Warrants linked to the price of oil? How do different market conditions affect the structural integrity of PCA? Can we use this information to develop a relative value model for the Brady market?

The PCA technique has been used by several authors to analyze financial data. To our knowledge, it has not yet been applied systematically to the Latin American Brady debt. For an overview of

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calendar days.

[4] Sovereign yield data was acquired from a proprietary market data source.

the literature on PCA, we refer the reader to Litterman and Scheinkman (1992) (U.S. term structure of interest rates), Avellaneda and Zhu (1997) (term structure of implied volatility of foreign-exchange options), Gourrieroux, et al (1997) for the CAC 40 Index and Laloux, et al (1999) for the Standard and Poor's 500 Index. Most of these studies use the entire available dataset to compute the empirical covariance, a procedure which is implicitly consistent with stationarity of the returns.[5] We believe that PCAs with long observation windows work best for the analysis of mature markets with stable economic cycles, which are consistent with the mathematical notion of stationarity. Quite in contrast, the Latin American Brady debt market is relatively young; it lacks the depth of the major G7 markets, and experiences sporadic periods of high volatility and regime shifts. In the case of Latin American Bradys, measuring market volatility on the basis of a single, long, observation window would underestimate the magnitude of market movements and fail to capture the market's reactions to dislocation events. Thus, we depart from the classical PCA paradigm by considering successive windows of observations and the corresponding covariance matrices rather than a single, long, observation window.

3 Analysis of Results: Order Out of Chaos

For comparative purposes, we begin with the analysis of the "static" PCA which uses the entire $5\frac{1}{2}$ year observation period. The corresponding PCA reveals three significant and easily identifiable principal components. Figure 2 provides a graphical representation of the direction and magnitude for the first, second and third principal components that explain on average 71%, 13%, and 6% respectively of the market's movement over the period. Under this static view of the market, the three principal components, explain almost 90% of the market variance.

The first principal component is such that all the coefficients have the same sign. It represents the volatility of the region as a whole. The coefficients of the principal component vector measure the

[5] The hypothesis of stationarity, or some "approximate" version, can be rejected in many cases. For example, Laloux *et al* reject the hypothesis of stationarity of returns of the the S&P 500 Index based on the spectral properties of the PCA.

typical relative changes in the value or yield of the corresponding bonds in response to a global shock to the region. Venezuelan Par and Discount Bonds are the most volatile during regional movements followed in order by Brazil, Argentina and Mexico. The differences in sensitivities between Par and Discount Bonds of the same issuing country is the direct result of a liquidity premium that the market places on Discount Bonds due to their smaller outstanding market size.

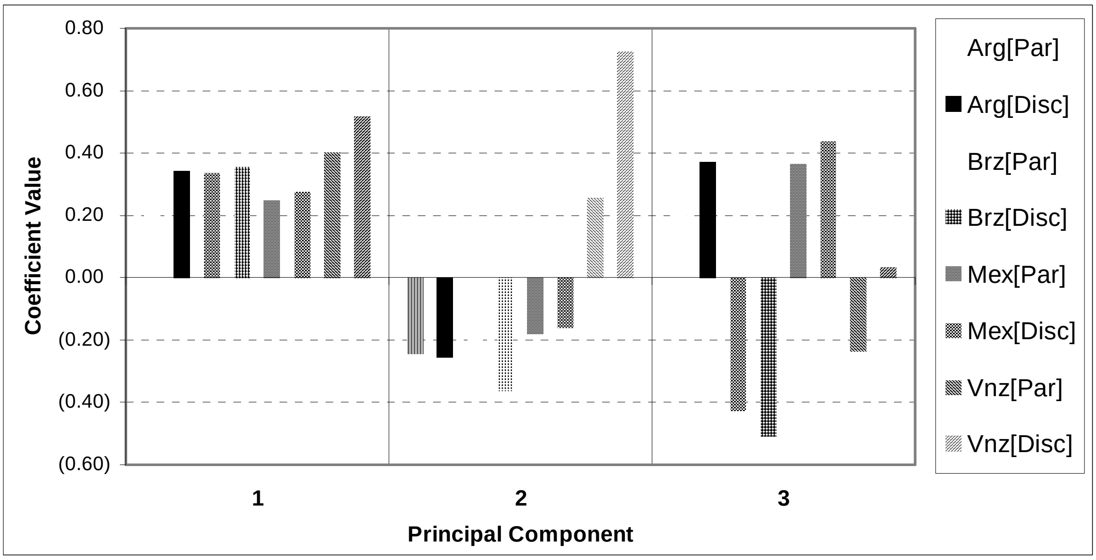
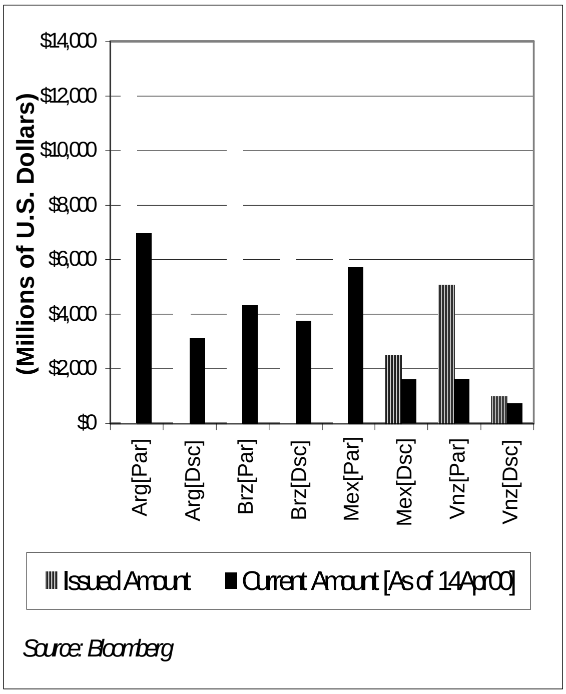
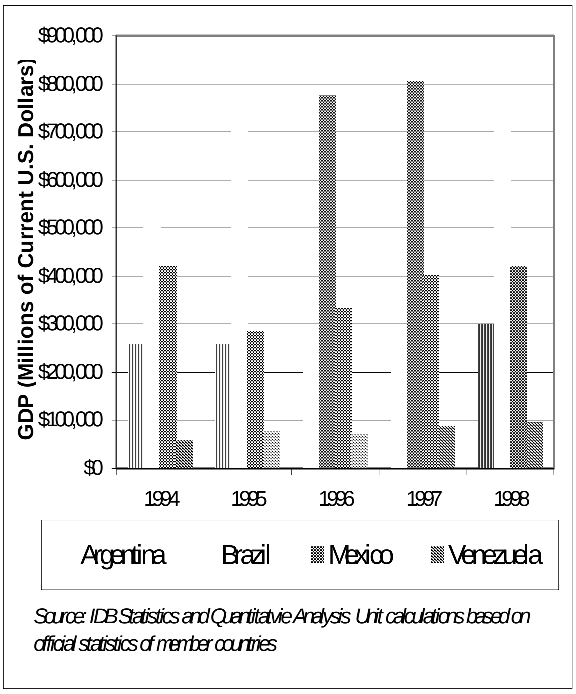


Figure 2: Static Principal Component Structures

It is clear from the data that the second principal component reflects the market's clear distinction between Venezuela and the other Latin American countries, Argentina, Brazil, and Mexico. We believe that this isolation, as well as the higher sensitivity of Venezuelan debt to regional movements, is due to three factors. First, the original and current amount of Brady Debt issued by Venezuela is considerably smaller than the other countries in the region. Second, as the figure below indicates, the Venezuelan gross domestic product is the smallest of the four countries, and third, it is also the least economically diverse in the region, depending on oil revenues for roughly 50% of GDP. The third principal component reflects primarily the relationship that exists between the three largest economies of Latin America: Argentina, Brazil and Mexico. Brazil is by far the largest, with a GDP approximately equal to the combined economies of Argentina and Mexico. We consider the second and third principal components as representative of the two primary regional relationships that exist within Latin America.





Figures 3 and 4: Original Issue and Current Outstanding Amount/Gross Domestic Product

To counter the deficiencies inherent to static analysis we present below the view of the market from a more dynamic point of view.

In a dynamic analysis framework we perform a sequence of PCAs over consecutive windows of 120 days. We shall refer to the fraction of variance attributed to the first component as the *coupling coefficient*.[6] The coupling coefficient represents the frequency with which the market moves as a single block under a typical shock.

[6] Thus, if $\lambda_1, > \lambda_2, \ldots\lambda_8$ are the eigenvalues, the coupling coefficient is given by the fraction

$$\frac{\lambda_1}{\sum_{i=1}^{8} \lambda_i}.$$

Throughout the five-year period, the coupling coefficient fluctuates significantly as the market takes views on the regional risk. We cannot however rely solely on the merits of the coupling coefficient to interpret the condition or state of the market. We believe that there is information in the structure of the first eigenvector as well. For each of the 12 windows, we provide a brief market synopsis that explains the economic events that are reflected in the PCA analysis. Summaries of these results are found in the following figures that graphically depict the time-evolution of the principal component structures and importances observed over the $5\frac{1}{2}$ year period.

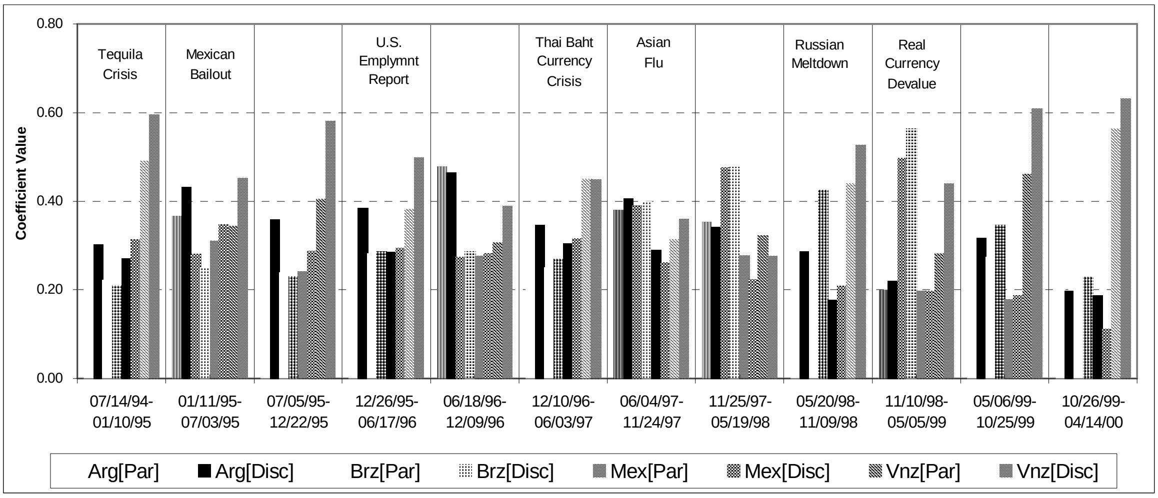


Figure 5: Migration of First Principal Component

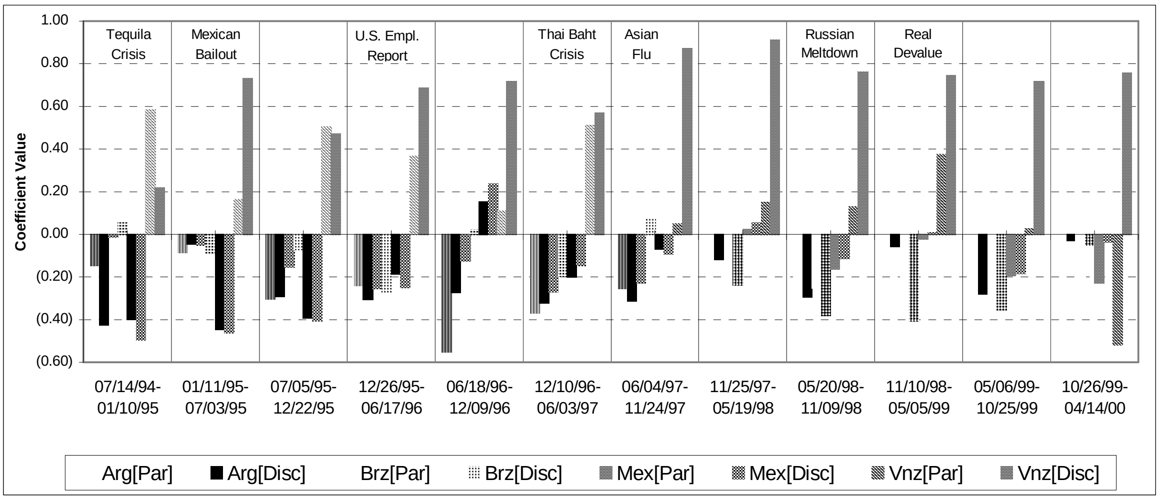


Figure 6: Migration of Second Principal Component

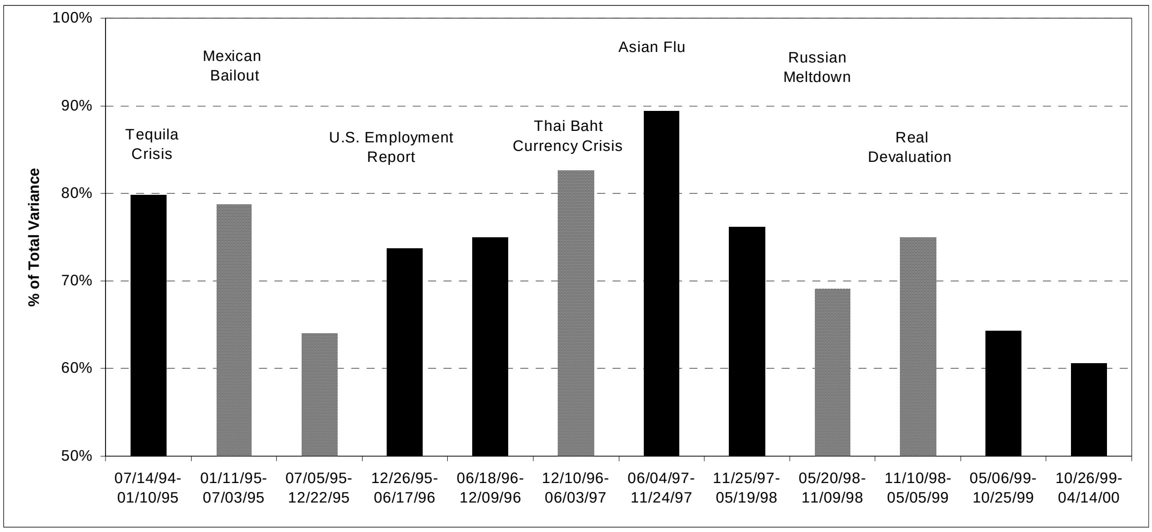


Figure 7: First Principal Component [Coupling Coefficient] Percentage of Total Variance

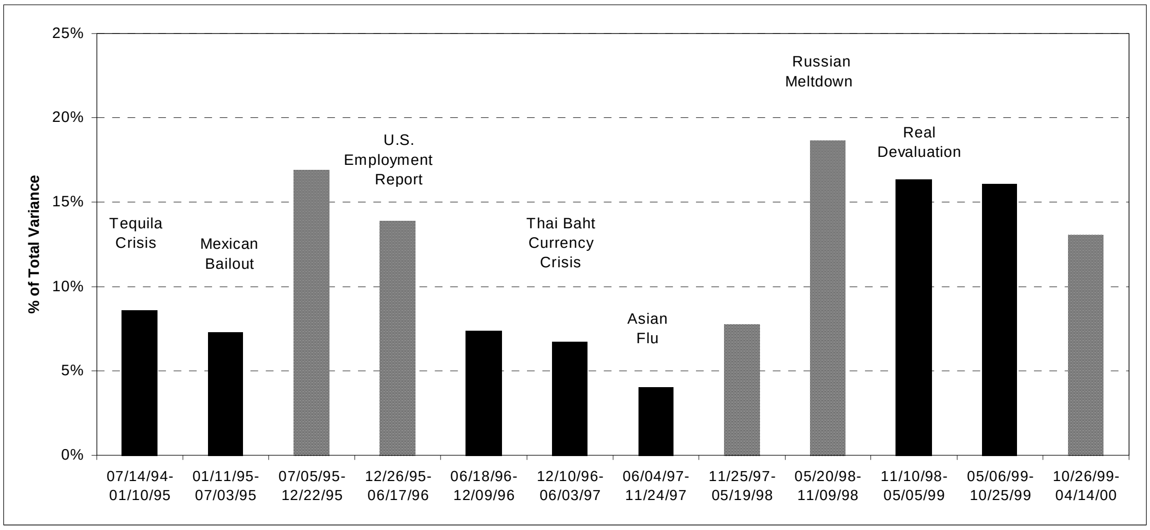


Figure 8: Second Principal Component Percentage of Total Variance

The first significant market dislocation event to impact Latin America within the observation period was the Mexican Peso devaluation (aka Tequila Crisis) in December 1994. In reaction to Mexico's currency devaluation the market demanded a substantial risk premium for holding Mexican Brady Debt. Driven by the fear that the single largest issuer of Brady Debt could potentially default, contagion quickly spread throughout the region as market participants subsequently penalized the other participating countries by also demanding a significant premium to hold their debt. During this period the coupling coefficient measured its largest value (80%) compared to all other endogenous shocks (i.e. shocks which can be traced to an event associated with the region). The first principal component coefficients of Venezuelan Par and Discount debt proved to be the largest (associated with more volatility) during the Tequila crisis by a significant margin compared to the other countries. This result is not surprising; Venezuela has the smallest economy and outstanding debt size of the four countries. The structure of the second principal component indicates that the market takes these two facts into account. Roughly 9% of the movement in the market over the period is attributed to the segregation of Venezuelan Debt from the rest of the region. Prior to the Tequila Crisis the market clearly distinguished to a much larger degree the Brady Debt of each country with differences in yields ranging from 75bp-600bp between countries. This marks the last time in which we would observe this level of yield differentiation between Latin debtor countries.

Mexico continued to reel from the effects of the Tequila Crisis in the following period. With cash reserves nearly depleted, Mexico was faced with the prospect of defaulting on it's short term local currency and dollar denominated debt obligations, Cetes and Tesobonos. Following an appeal to the United States of America arrangements were made to extended a special line of credit from the U.S. Treasury, secured by petroleum reserves. This effectively amounts to a bailout that enabled Mexico to meet it's debt obligations. Financial institutions, primarily in the United States, had significant exposure to Mexican debt and neither they nor the U.S. Government wanted a replay of events that had transpired in the 1980's. This had a profound effect on the market. First, the United States, in recognizing the relative importance of the region, placed an implicit floor on the value of the debt. Second, investors would henceforth place more emphasis on the region as whole than on individual countries. With a coupling coefficient nearly as high during the Tequila Crisis, 79%, the Brady Debt moved in "lockstep", as the market reacted to the crisis and the Mexican "dollars-for-oil" negotiations. These events were also responsible for triggering a significant structural change in the second principal component from the previous period. The dominant intra-regional relationship, accounting for $7\frac{1}{4}\%$ of the market's activity, almost exclusively reflects the movement of Venezuela versus Mexico while relegating Argentina and Brazil as intra-regional bystanders.

The period covering July 1995 through December 1995 was relatively benign as the market continued to digest the events of the previous six months. The most important development with respect to this period was the emergence of the second principal component as a significant factor in explaining the movement of the market. It is noteworthy that the second principal component always existed but only in a limited capacity until this period. Accounting for almost 17% of the market variance in this period, the second principal component isolated the relative yield spread widening of Venezuela versus Argentina, Mexico and to a lesser degree Brazilian Par Bonds.

During 1996, corresponding essentially to the fourth and fifth windows, we witness the beginning of a $2\frac{1}{2}$ year period of historically unprecedented investor demand of Latin American debt. The March 1996 U.S. Employment Report provided the only significant shock during the period as the market was caught off guard when $700,000$ new jobs were added to the U.S. economy, signaling the possibility of inflation. Argentine Brady Debt clearly had the most adverse reaction to the report, due to Argentina's dollarization program that directly pegs their currency to the U.S. dollar. Reflective of both the employment shock and overall investor confidence in the region, the coupling coefficient measured 74%, representing an increase of 10% over the previous period and maintained this level for the latter half of 1996. Progressing from the fourth to fifth periods saw a significant change in the structure of the second principal component. Up until the second half of 1996, the second principal component captured the market variance attributed to Venezuela versus Argentina, Brazil and Mexico. This relationship shifted to reflect a "north versus south" regional movement between Venezuela and Mexico on the one hand versus Argentina and Brazil on the other. Clearly, with the former two countries heavily dependent on oil revenues it is worth investigating the potential impact crude oil prices may have on the relevance of this structure (see Appendix B). We believe that the decrease in the second principal component from 14% to 7% between the first and second half of 1996 can be attribute to the structural change that occurred, i.e. a shift to a more "monolithic" perception of the region (and its hemispheres).

In 1997, two exogenous shocks originating in Asia provided the two largest coupling coefficient measurements during the entire observation period. These became known as the Thai Baht Currency Crisis and the Asian Flu. We characterize these events as acute shocks having a limited, negative impact on the Latin Brady market for a relatively abbreviated time period. As the first significant external shock to the market, the Thai Baht Currency crisis in late spring of 1997 further solidified the importance of the coupling coefficient primarily at the expense of a shift of variance from the third and higher order components. The proportion of market variance attributed to the second principal component decreased slightly however. The structure of the second component reverted back to an intra-regional volatility driven by Venezuela on one side and Argentina, Brazil, and

Mexico on the other. Following a small respite of six months, the Asian Flu hit an already shaky investor confidence base in the region as Par and Discount Brady Bonds moved together almost 90% of the time. The presence of these two shocks reinforced the importance of a coupling coefficient. Before the crisis the first principal component explained 75% of the market's movement, a relatively high correlation state, before "jumping" to a more extreme state as a result of the two external shocks. This demonstrates the "herd mentality" of investing within the region. With such a significant proportion of the market movement accounted for by the first component the second principal component became immaterial, just below 4%.

The first half of 1998 will be remembered as the "calm before the storm". Investor participation in the market was at full tilt with yields on Par and Discount Bonds reaching historically their lowest levels. The coupling coefficient returned to a more modest level of just over 75% and the second principal component doubled in importance to roughly 8%. However the second principal component changes structure once again, isolating the relationship of Venezuela versus Argentina and Brazil with Mexico effectively non-participants.

The catalyst for what will be remembered as one of the most dramatic financial events of the last decade was Russia's default of local currency external debt. The near-collapse of Russia's economic framework was triggered by the Ruble currency devaluation in August of 1998 as investor confidence in the ability of the Russian government to meet its obligations precipitously eroded over the summer. By early fall Russia had indeed not only defaulted on local currency debt or GKO's[7] but also on debt issued during the Soviet-era. This event had significant repercussions throughout the financial world and prompted a strong " flight to quality" as investors hoarded the government debt of the United States and European governments. Latin American Par and Discount Debt, which were yielding $8-9\%$ just a few months before, now traded with yields in the range of $12-13\%$. Of particular interest from the PCA viewpoint is the fact that the coupling coefficient decreased from

[7] Gosudarstvennyie Kratkosrochynie Obligatsii's or GKO's are sovereign bonds denominated in Russian Rubles. Between February 1998 and June 1998 the Russian government issued approximately $65 Billion (RUB) with 50-Year Maturities. Various plans were proposed by the International Monetary Fund to restructure the defaulted debt but have been rejected by both Russia and the creditors.

the previous period to 69% from 76% and the second principal component registered its peak variance of 19%. Therefore, despite the widening of spreads, the coupling coefficient did not rise to the levels of 1997. We conjecture that the close economic relationship that exists between Mexico and the United States and Argentina's U.S. Dollar-linked economic policy underlies this phenomenon. For example, the data shows that the yields for Brazilian and Venezuelan debt increased significantly more than those of Argentina and Mexico. During this period of turmoil, the large importance of the second principal component reflected the risk premium demanded for holding Venezuelan debt relative to the rest of the region. We also believe that when the Brady Bond market came under duress in the Fall of 1998, the lack of outstanding issuance size of the Venezuelan Brady Debt and its smaller economy relative to the other countries were the significant factors that affected the increase in importance of the second component.

Tepid investor confidence returned to the region following the Russian Meltdown but it was short lived, as Brazil devalued its currency in January 1999. This resulted in an increase in the coupling coefficient from just below 70% to 75% coming primarily as a result of a shift of market variance from the second principal component. The latter decreased 2.5% to 16%. Brazilian Par and Discount Debt experienced the largest absolute and relative change in the first principal component coefficients from the previous period and are primarily responsible for the increase in variance over the previous periods coupling coefficient. The impact of the Brazilian devaluation was also felt at the level of the second principal component. The market clearly focused on the relationship between Brazil and Venezuela during this period. The market perceived, on a relative basis, that the risk inherent it taking a position in Brazilian debt was greater than that of Venezuela. This was reflected in the increased yield spread for investing in Brazil.

The last half of 1999 and the first half of 2000 see the decrease in importance of the coupling coefficient, measuring 64% and 60% respectively, levels not seen since the second half of 1995. Stability and investor confidence gradually returned to the market following the Brazilian devaluation. However it is quite clear that the Par and Discount debt of the respective countries are viewed more on an individual basis than as a collective whole. The PCA model confirms this with the first significant appearance of the third principal component. Unfortunately, drawing any long term conclusions based on the contribution of the third principal component is difficult because it did not play a significant role beforehand. What we can confirm however is a diffusion of market variance from the first principal component to the third and higher order components. The second principal component structurally reverts to its historical form but decreases slightly in significance during the first half of 2000.

Associating the results derived from the PCA analysis to the behavior of the market over the past five years certainly promotes a sense of realism and better understanding of the Latin American Brady Bond debt market than what was observed under a single period static scenario. With all of this information now available we can turn our efforts towards answering some of the questions that were posed beforehand.

4 Principal Component Analysis: The Structure Revealed

4.1 Coupling Coefficient

The coupling coefficient (percent of variance explained by the first component) warrants closer attention, due to its importance in understanding "regional risk". There are three regimes that measure the degree to which the Brady Bond debt moves collectively in a given period: weak, strong, and extreme. To gauge the relative strength of the coupling, we focus on (i) market direction or state (ii) volatility of the sovereign yield changes for the individual par and discount bonds (see figure below), (iii) type of the dislocation event and (iv) the absolute level of the coupling coefficient. [8] [9]

[8]The variance or volatility in this context is associated with the individual par and discount bond and should not be confused with the variance applicable to a particular principal component. Variance in this context is used interchangeably with market volatility. The higher the variance the greater the volatility in stripped yield changes or movements.

[9]There are two types of dislocation events or shocks, acute and protracted. Acute shocks are short in duration and do not have a significant impact on the long-term behavior of the market. Protracted shocks signify a fundamental change in the direction of the market; the effect of which may impact the market for a significant period of time.

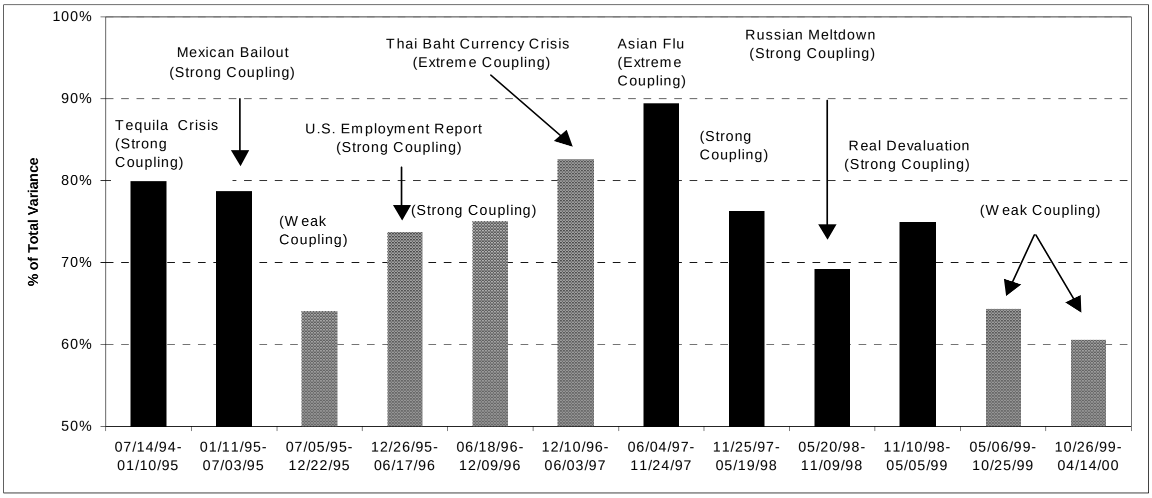


Figure 9: Coupling Coefficient Regimes

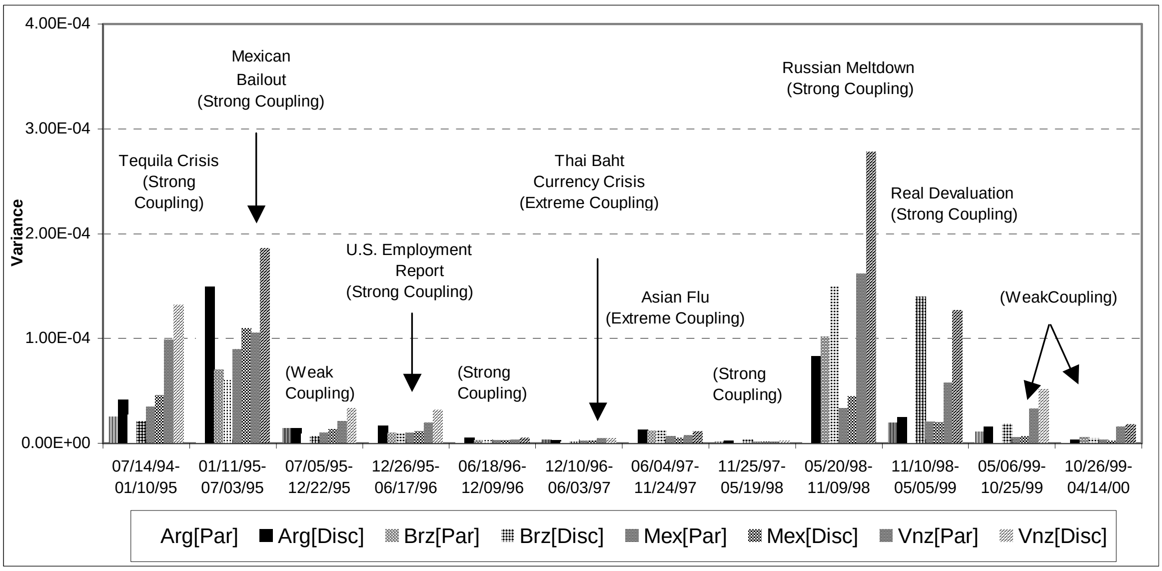


Figure 10: Par and Discount Bond Sovereign Yield Spread Variance Levels

A coupling coefficient of less than 65% indicates that the Latin American Brady Bond debt is not well correlated. A weak coupling is indicative of "range bound" market behavior, where more emphasis is placed on the individual country risk as opposed to regional risk. Weak couplings tend to follow protracted dislocation events and represent a period in which the market re-establishes an equilibrium as it assesses the impact of the shock.

At the opposite end of the spectrum, a regime is considered extreme when the coupling coefficient exceeds 80%. During extreme coupling regimes, an almost one-to-one relationship exists in the directional movement of the respective Brady Bond debt. Market participants disregard the risk associated with investing in an individual country and focus almost exclusively on the aggregate risk of the region. There are two fundamental factors that we believe are responsible for the market to achieve such a high level of correlation; first, an acute exogenous shock to the region and second, the state of the market *preceding* the dislocation event. An acute exogenous shock, such as the Asian Flu or Thai Baht currency crisis, presents the market with a dilemma: unable to directly attribute the shock to a regional source, the market assumes a defensive posture that indiscriminately impacts the region. Evidence of this is seen in the relatively low volatility of the sovereign yield changes as the market "jumps" in response to the shock. This effect is further magnified by an already strong directional correlation in the market leading up to the shock that ultimately results in an extreme level of coupling.

Strong coupling states are observed when the coefficient lies between $65\% - 80\%$ and indicates that a high degree of correlation exists in the aggregate movement of the Brady Bond debt. A strong coupling state can exist in two very different market and volatility environments, namely, (i) in the presence of a protracted dislocation event when market volatility reaches extreme levels, such as the Tequila Crisis and Real currency devaluation, or, (ii) during periods in which market demand for Latin American debt reaches (equally) euphoric levels under low volatility conditions.

4.2 Structural Integrity of the Principal Components

On the whole, when we consider the three primary Latin American countries, Mexico has the smallest coefficients in first principal component elements. This reflects a greater perceived stability on the part of the market, in relation to the other countries. In large part, this is due to the close relationship to the United States of America and the effectiveness of it's fiscal and monetary policy. The recent upgrades in credit worthiness of Mexican Brady Debt may in part explain why the coupling coefficient for the past year remains at such low levels. Up until November 1997, Argentine Brady Debt proved to be much more sensitive to regional movements than Brazil. Since that point however the exact opposite has been true. The relative stability of Argentine debt versus Brazilian debt is believed to result from Argentina's U.S. dollarization program.

The second principal component specifically measures the dominant intra-regional relationship between the par and discount bonds of the participating countries. Unlike the first PCA, the second PCA is structurally dynamic and can vary significantly from period to period. In particular it would have been counterintuitive for the structure of the second principal component to remain similar given the economic events which affected the region. As the table below indicates, the most frequently observed structures isolate the movements between the Par and Discount debt of Venezuela versus the debt of Argentina, Brazil and Mexico[10].

Second PCA Structure	Number of Observations
Venezuela vs. Argentina/Mexico/Brazil	5
Venezuela vs. Argentina/Brazil	2
Venezuela vs. Argentina/Mexico	1
Venezuela vs. Brazil or Mexico	1 each

Table 1: Second PCA Dynamic Structures

Clearly, the second PCA structure is defined by Venezuela versus Argentina, Brazil, and Mexico. Periods in which the relationship did not hold true are those observation windows that contained a dislocation event originating within Latin America or during a period in which the strength of the coupling coefficient was either strong or extreme.

As we had noted earlier, the table accounts for only ten out of the twelve periods. We have

[10]The number of observations recorded within the table represents only 10 out of the possible 12 trading period windows. One of the principal component structures failed to meet certain stability requirements and was therefore rejected.

excluded those second PCA structures from the analysis due to the statistical instability of the coefficients. In particular the asymptotic standard errors, which serve as approximate estimates of the variability in the coefficients, were well above acceptable limits. (We refer the reader to Appendix D which provides a more in depth explanation into statistical stability of PCA as it relates to the analysis)

4.3 Contrast of Static and Dynamic PCA

If we consider the results of static PCA as representative of a long-term statistical market mean of sorts and those of dynamic PCA representing short-term market behavior there exists significant differences between the two. The most striking example of which is the behavior of the coupling coefficient. On the basis of the criteria used to determine the coupling coefficient regime we would categorize the static coupling coefficient as marginally strong indicating that the market moves in a relatively high correlated manner with moderate volatility. It is quite clear by this juncture that both the coupling coefficient and market volatility in a dynamic PCA environment behave with significant variability. The former moves between an upper boundary of 90% and lower boundary of 60%, while the latter, on several occasions, exhibits sustained periods of high volatility. (For comparative purposes we contrast the first PCA proportion of variance for static, dynamic, and what we refer to as continuous in Appendix C.) On this basis alone we conclude that there is a high degree of risk in over or under-estimating the behavior of the market if considering only static PCA. This fact is further reinforced when we make comparisons involving the first principal component coefficients or structure of the second PCA. With respect to the latter, it is evident that the statistical significance of the second PCA is not always material nor does it consistently reflect a static structure. The effects of dislocations are muted under static analysis since we are unable to ascertain their impact on a given country or the region as a whole.

Three factors were utilized to model the behavior of the market under static analysis, however, stability limitations or the lack of structural consistency limited the dynamic PCA to the first two principal components (See Appendix D). The two-factor dynamic PCA model accounted for on average 85% of the proportion of market variance with upper and lower limits of 70% and 93% respectively. This compares to a 90% proportion of market variance under a three factor static PCA model. We contend that although one less factor is utilized dynamic PCA provides a much more realistic interpretation of the market's behavior at what we believe is a relatively small statistical cost.

For those readers interested in viewing the results for the third and higher order components we have included in Appendix E the covariance matrices for each of the twelve trading period windows as well as the covariance matrix used for static analysis.

5 Conclusions and Further Application for PCA

The analysis of this paper took the covariance of bond prices/yields in the Latin American Brady market and analyzed its structure dynamically. Particular attention was given to the changes in the relative importance of the factor in the presence of the different macro-economic shocks that affected the region in the past six years. We believe that PCA has helped to determine the variation of importance in the components, or "correlation risk". In particular, the consistent appearance of Venezuela as the primary intra-regional source of risk follows clearly from our analysis. This result alone should have interesting implications in terms of asset-allocation strategies for the region. It will be interesting to study the future dynamics of the PCA as new events unfold.

Principal Component Analysis looks at the market in terms of volatility and correlation risk, as opposed to the more conventional measure of "total return". One interesting statistic which emerges from PCA is the coupling coefficient, or percentage of variance explained by the first component. By observing the variations of the coupling across time, we can form a dynamic picture of how the market viewed the "regional" risk in the past 6 years. A final important application of factor analysis, which could be the focus of future research, is to use these results to construct credit risk models for pricing default protection, collateralized bond obligations and other credit-sensitive instruments.

References

[1] Anderson, T.W. (1963): Asymptotic theory for principal component analysis. *Annals of Mathematical Statistics*, **34**, 122-148.

[2] Avellaneda, M. and Zhu, Y (1997).: An E-Arch Model for the Term Structure of Implied Volatility of FX Options. *International Journal of Theoretical and Applied Finance*

[3] Devlin, S.J., Gnanadesikan, R., and Keramidas, E.M. (1987): A Graphical Procedure for Comparing the Principal Components of Several Covariance Matrices. *Communications in Statistics B- Computation and Simulation*, **16**, 161-191

[4] Fluery, B. (1988): *Common Principal Components and Related Multivariate Models*. John Wiley & Sons, New York

[5] Fuller, W. (1996): *Introduction to Statistical Time Series*. John Wiley & Sons, New York

[6] Gourrieroux, Ch., Scaillet. O., and Szafarz, A. (1997): Econometrie de la Finance. *Ed. Economica*, Paris

[7] Krzanowski, W.J. (1984a): Sensitivity of Principal Components. *Journal of the Royal Statistical Society*, ser. B, **46**, 558-563.

[8] Laloux, L.,Cizeau, C., Bouchaud, J-P., and Potters, M. (1999): Noise Dressing of Financial Correlation Matrices. *Physical Review Letters 83*, **7**, 1467

[9] Litterman, R. and Scheinkman, J. (1991): Common Factors Affecting Bond Returns. *Journal of Fixed Income*, **June**, 55-61.

[10] Schloerb, F.P., Heyer M.H. (1997): Application of Principal Component Analysis to Large-Scale Spectral Line Imaging Studies of the Interstellar Medium. *The Astrophysical Journal*, **475**, 173-187.

[11] Strang, Gilbert (1988, 3rd ed.): *Linear Algebra and Its Applications*. Saunders College Publishing, New York

[12] *Country Analysis Briefs: Central & South America.*

http://www.eia.doe.gov/emeu/cabs/venez.html (13 January 2000)
http://www.eia.doe.gov/emeu/cabs/mexifull.html (01 February 2000)

6 Appendix A: PCA Mathematical Framework

We start with an 8-dimensional vector; $\left(\delta_p^{(1)}, \delta_{(p)}^{(2)}, \ldots, \delta_p^{(8)}\right)$, where each $\delta_p^{(n)}$ represents a sovereign yield change for a par or discount bond of Argentina, Brazil, Mexico or Venezuela at a particular observation point. We represent the time-series market data by the 8×1 vector at each observation point p

$$\mathbf{X}_p = \left[\delta_p^{(1)}, \delta_p^{(2)}, \ldots, \delta_p^{(8)}\right].$$

We consider the empirical covariance matrix of the vector X, defined as $\mathbf{S} = (s_{ij})$, with

$$s_{ij} = \frac{1}{T-1} \sum_{t=1}^{T-1} \left(\delta^{(i)}(t+1) - \delta^{(i)}(t)\right) \left(\delta^{(j)}(t+1) - \delta^{(j)}(t)\right),$$

where $1 \leq i, j \leq 8$ and t ranges over the number of days observed. The matrix $\mathbf{S}$ is symmetric and non-negative definite. We can therefore use the spectral decomposition theorem[11] to obtain

$$\mathbf{S} = \mathbf{B} \mathbf{\Lambda} \mathbf{B}' = \sum_{j=1}^{p} \lambda_j \mathbf{b}_j \mathbf{b}_j'.$$

Here $\mathbf{B} = (\mathbf{b}_1, ..., \mathbf{b}_8)$ represents an orthogonal matrix having the principal components as its columns. The matrix $\mathbf{\Lambda}$ is diagonal; its entries are the characteristic roots sorted in decreasing order, $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_8$. This decomposition allows us to interpret the vector of yield changes $\mathbf{X}$ as a sum of random orthogonal components, i.e.,

$$\mathbf{X} = \overline{\mathbf{X}} + \sum_{j=1}^{p} \sqrt{\lambda_j} \mathbf{b}_j n_j,$$

[11] Without proof, we present the diagonalization and spectral decomposition for symmetric matrices from Fluery (1988, p.217). Let A denote a real symmetric matrix of dimension $p \times p$. Then there exists an orthogonal matrix B and a diagonal matrix Λ such that $B'AB = \Lambda$. If the characteristic roots A of are all distinct, the matrix is B uniquely defined up to the multiplication of columns by -1 and the permutation of columns. The columns of B $(say, b_1, \ldots, b_p)$ are the characteristic vectors of A, normalized such that $b_j' b_j = 1$ $(j = 1, \ldots, p)$, and the diagonal elements of Λ are the associated characteristic roots.

with n_j being uncorrelated random variables with mean zero and variance one.

7 Appendix B: Crude Oil and Brady Bond Debt

Earlier in the analysis we speculated that one of the factors that may have resulted in a second principal component that divided Venezuela and Mexico from Argentina and Brazil was the former two countries' heavy dependence on oil revenues. In Venezuela, holding the largest oil reserves in the western-hemisphere, oil accounts for approximately three-quarters of total exports, half of government revenues, and about one-half of GDP[12]. Next to Venezuela, Mexico holds the largest oil reserves in the western-hemisphere. Although not as heavily dependent as Venezuela oil still plays a significant role in the Mexican economy accounting for 7% of total exports and 33% of government revenues[13]. Given the substantial economic dependence on crude oil and the Value Recovery Rights linked to the price of oil embedded in the structure of Mexican and Venezuelan Par and Discount Debt one is easily tempted to conclude that crude oil prices would have an impact on the price of the Brady Bond debt. In an attempt to confirm this we produce a covariance matrix on a price change basis for Venezuelan and Mexican Par and Discount Debt together with the three Latin American crude oil grades, Maya, OlMeca and Isthmus as well as West Texas Intermediate. The covariance matrix, presented on the following page, clearly indicates that the price of crude oil does not impact the behavior of the Venezuelan or Mexican Par and Discount Debt. Not entirely convinced of the results we approached the question more on the basis of options theory. In effect the market does not take into consideration the intrinsic value of the Value Recovery Rights even though the underlying Brady Bond may not mature for 20 years or so. Simply stated, the exercise or strike price is so far out of the money that the majority of investors effectively ignore the Value Recovery Rights option.

[12] Energy Information Administration, January 2000
[13] Energy Information Administration, February 2000

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]	WTI	OlMeca	Isthmus	Maya
Arg[Par]	**0.6562**	0.6899	0.5092	0.5173	0.4980	0.4845	0.4619	0.3843	(0.0045)	(0.0110)	0.0102	(0.0014)
Arg[Disc]	0.6899	**0.9793**	0.5732	0.6715	0.5577	0.6494	0.4908	0.4815	0.0019	(0.0006)	0.0178	0.0013
Brz[Par]	0.5092	0.5732	**0.6058**	0.5853	0.4314	0.4322	0.4237	0.3682	(0.0046)	(0.0092)	0.0096	(0.0034)
Brz[Disc]	0.5173	0.6715	0.5853	**0.7579**	0.4415	0.5114	0.4381	0.4332	(0.0042)	(0.0116)	0.0080	(0.0046)
Mex[Par]	0.4980	0.5577	0.4314	0.4415	**0.5714**	0.5505	0.4053	0.3388	0.0075	0.0024	0.0052	0.0081
Mex[Disc]	0.4845	0.6494	0.4322	0.5114	0.5505	**0.7196**	0.3880	0.3864	0.0058	0.0031	0.0011	0.0071
Vnz[Par]	0.4619	0.4908	0.4237	0.4381	0.4053	0.3880	**0.6621**	0.4969	0.0099	0.0065	0.0098	0.0060
Vnz[Disc]	0.3843	0.4815	0.3682	0.4332	0.3388	0.3864	0.4969	**0.6703**	0.0078	0.0063	(0.0034)	0.0072
WTI	(0.0045)	0.0019	(0.0046)	(0.0042)	0.0075	0.0058	0.0099	0.0078	**0.2006**	0.1629	0.1586	0.1130
OlMeca	(0.0110)	(0.0006)	(0.0092)	(0.0116)	0.0024	0.0031	0.0065	0.0063	0.1629	**0.2049**	0.1680	0.1131
Isthmus	0.0102	0.0178	0.0096	0.0080	0.0052	0.0011	0.0098	(0.0034)	0.1586	0.1680	**0.5310**	0.1117
Maya	(0.0014)	0.0013	(0.0034)	(0.0046)	0.0081	0.0071	0.0060	0.0072	0.1130	0.1131	0.1117	**0.1123**

Table 2: Brady Bond Debt with Crude Oil

8 Appendix C: Continuous PCA

A natural extension of "dynamic" PCA is to view the market on a "continuous" PCA basis that consists of utilizing covariance matrices on a time series consisting of a continuously rolling 120-Day observation window. Interpretation of the results conducted on a continuous basis is far more difficult and involved than those derived under dynamic analysis; the number of matrices alone exceeds 1,000. There are however intermediate results that we can utilize that enhance dynamic analysis. Since a large portion of the research focuses on the behavior of the first principal component or coupling coefficient we present below the proportion of variance attributed to this principal component based on three different time-series windows, static, dynamic and continuous. Comparatively, dynamic analysis provides a rough approximation to the results derived on a continuous time basis. We expected the continuous time series results to exhibit higher and lower levels for the proportion of variance for the first principal component and support our conjecture that the degree to which the Brady Bond Debt market behaves would be misrepresented on the basis of a long term time series. This is true if we consider the results from the static analysis to represent a statistical mean of sorts.

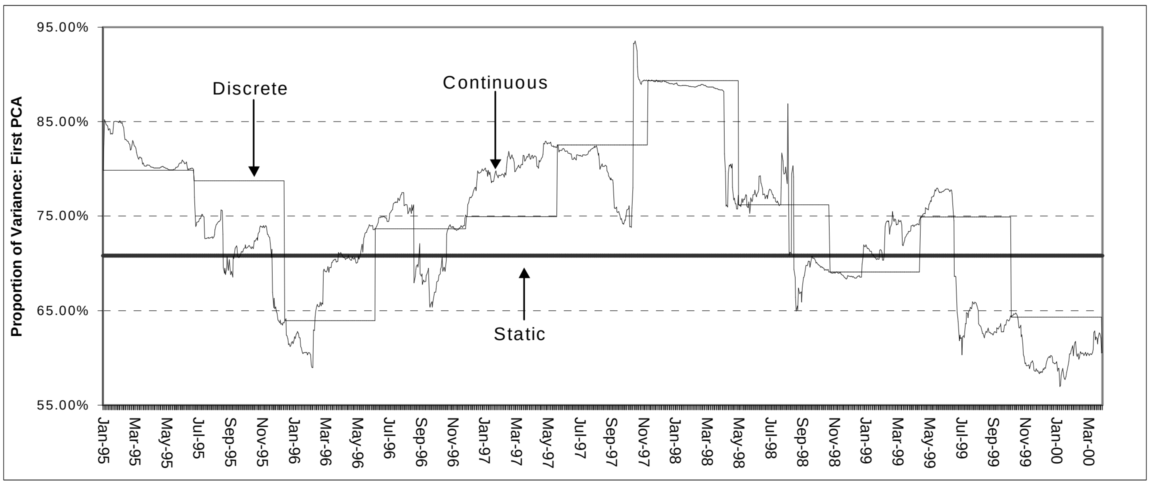


Figure 11: Comparison of Static, Dynamic and Continuous 1st PCA % of Total Variance

One of our objectives for utilizing a continuous basis was for the purpose of extracting information from the PCA that would provide an indication of the future behavior of the market. We were however unable to directly establish a relationship between the PCA results and the direction of the market. However, we also believe that the factor analysis derived from the continuous PCA results could prove to be a integrable part of constructing models for the purposes of relative value or risk assessment.

9 Appendix D: Stability of Principal Components

Before interpreting principal components it is highly recommended that the stability of the component coefficients is established because the estimated coefficients are subject to some sampling variability. We utilize the asymptotic normal theory approach proposed by Fluery (1988) which consists of determining the log-likelihood ratio test statistic for the hypothesis of sphericity and the asymptotic standard errors for the characteristic vector elements. Before calculating the asymptotic standard errors of the coefficients we first confirm that the associated characteristic roots λ_h of the covariance matrix Ψ are distinct such that $\lambda_{h-1} > \lambda_h > \lambda_{h+1}$. We utilize the sphericity test statistic:

$$S(l_{h-1}, l_h) := 2n \log \frac{l_{h-1}+l_h}{2\sqrt{l_{h-1}l_h}} \ ,$$

(where n = number of observations and l_h are the maximum likelihood estimates of λ_h) which is asymptotically distributed as chi square on 2 degrees of freedom under the null hypothesis.

>From the asymptotic theory for sample principal components (see Fluery 1988, chapter 2)[14] it is clear that under the assumption that the characteristic roots are distinct, asymptotically,

$$\sqrt{n}\,(b_{mh} - \beta_{mh}) \sim N\,(0, v_{mh})$$

where v_{mh} is the (m, m) element of

[14]Let S denote a random symmetric $p \times p$ matrix, distributed as $W_p\,(n, \Psi/n)$, where Ψ is positive definite and symmetric. Let $S = BLB^{\prime}$and $\Psi = \beta\Lambda\beta^{\prime}$be the spectral decomposition of S and Ψ, $L = diag\,(l_1, \ldots, l_p)$ and $\Lambda = diag\,(\lambda_1, \ldots, \lambda_p)$. Assume that all λ_j are distinct. Then

a. The asymptotic distribution of

$$\sqrt{n}\begin{bmatrix} l_1 - \lambda_1 \\ \vdots \\ l_p - \lambda_p \end{bmatrix}$$

as n tends to infinity is p-variate normal with mean zero and covariance matrix $diag\left(2\lambda_1^2, \ldots, 2\lambda_p^2\right)$, and the l_j are asymptotically independent of B.

b. The asymptotic distribution of $\sqrt{n}vec(B - \beta)$ is p^2-variate (singular) normal with mean zero and covariance matrix V_B

$$\sum_{\substack{j=1 \\ j \neq h}}^{p} \theta_{hj} \beta_j \beta_j^{'} \text{ where } \theta_{hj} = \begin{array}{cc} 0 & h = j \\ \frac{\lambda_h \lambda_j}{(\lambda_h - \lambda_j)^2} & h \neq j \end{array}$$

If the characteristic roots λ_j are close, then the elements of the associated characteristic vectors may have large variability.

For large n, $b_{mh} = N(\beta_{mh}, v_{mh}/n)$, we estimate the variance by replacing the parameters in the above equation by their sample counterparts. The asymptotic standard error of b_{mh} is defined as

$$s\left(b_{mh}\right) = \left[\frac{1}{n} l_h \sum_{\substack{j=1 \\ j \neq h}}^{p} \frac{l_j}{(l_j - l_h)^2} b_{mj}^2 \right]^{1/2}$$

We apply the sphericity test separately to each of the twelve observation windows. When we compute the respective asymptotic standard errors, we find that the first principal coefficients are very stable for each of the twelve periods. Variability in the coefficients becomes greater for the higher order eigenvectors, as the separation between the eigenvalues decreases. This relationship is evident when we measure the second and third components for statistical stability. The results of the sphericity test statistics for the second and third components are presented below. In the case of the second component sphericity cannot be rejected in one-fourth of the observations when we use a chi square distribution with 2 degrees of freedom at a significance level of 5%. In the case of the third component sphericity cannot be rejected for roughly one-third of the observation windows. For two out of the three cases where the second principal component failed to "pass" the sphericity test the asymptotic standard errors of the coefficients, see figure below, are significantly above what we considered an acceptable level, $s\left(b_{mh}\right) \leq 0.15$[15]. We conclude that these second order principal components are effectively uninterpretable. The instability of the third principal component supports our decision to only attache significance to the first two components in our dynamic PCA. We believe that the small sample size (120 days) precludes us from drawing conclusions about the structure of third or higher order components in the dynamic setting.

[15] Certainly the smaller the asymptotic standard error the greater the stability of the coefficients. However there is no set statistical limit to determine what is considered to much variability in the coefficients.

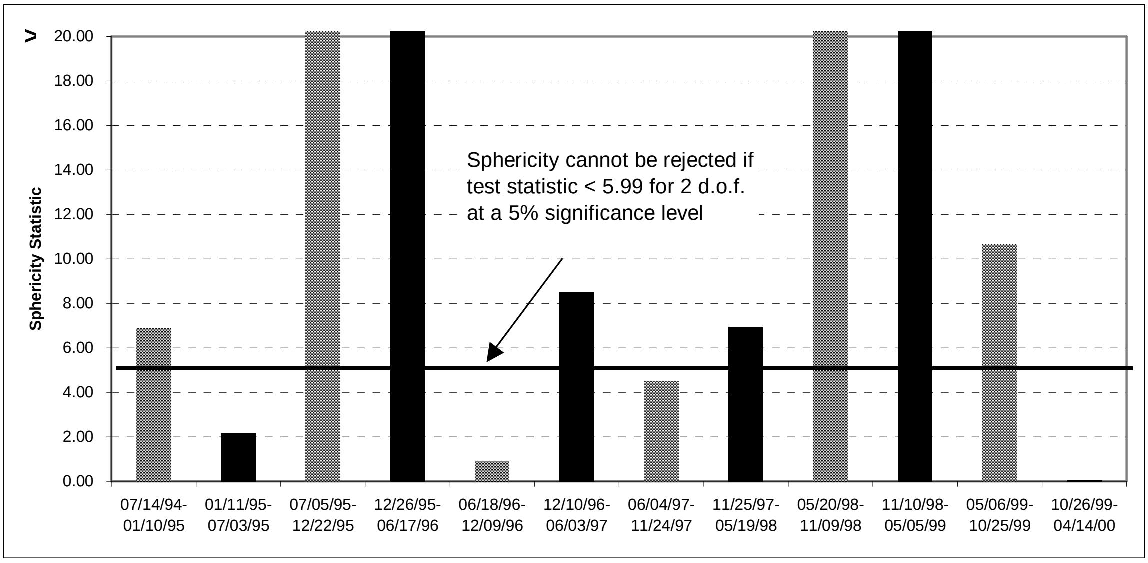


Figure 12: Sphericity Test Statistic for Second Principal Component

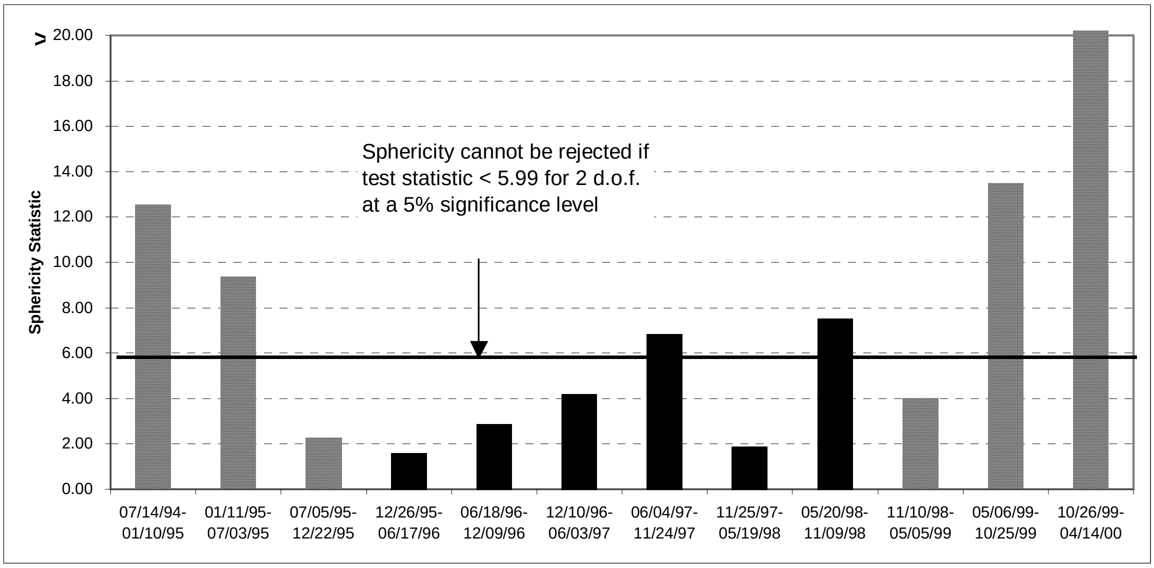


Figure 13: Sphericity Test Statistic for Third Principal Component

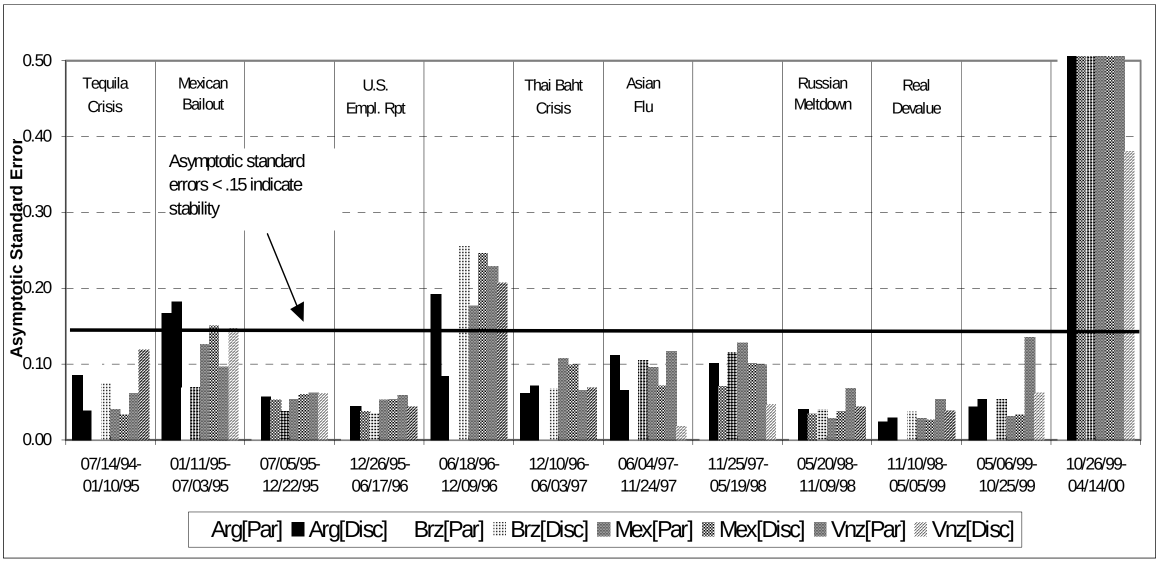


Figure 14: Asymptotic Standard Errors of Second PCA Coefficients

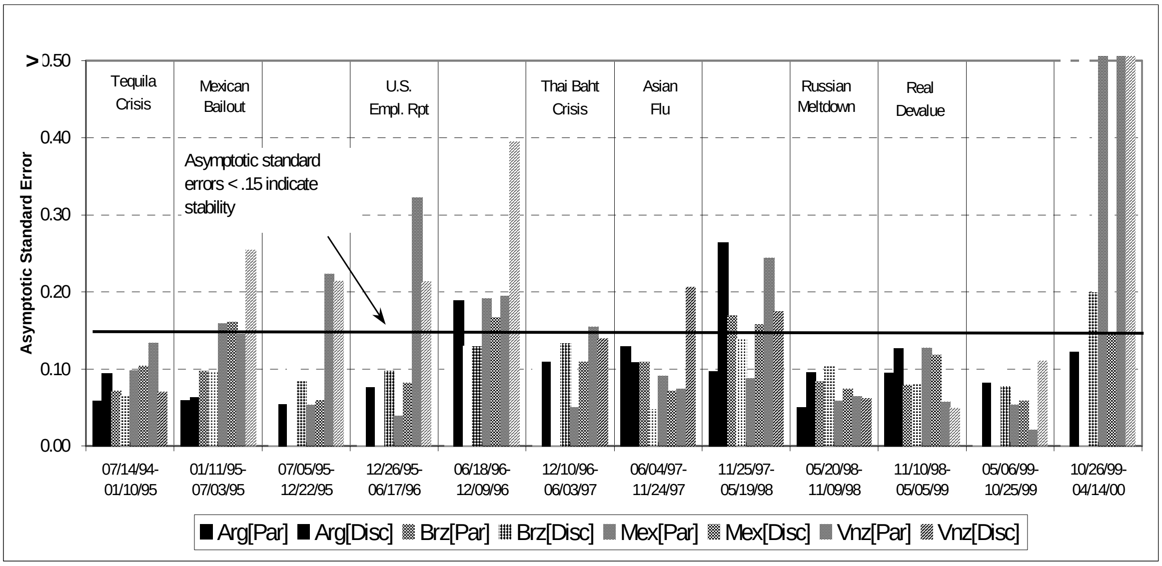


Figure 15: Asymptotic Standard Errors of Third PCA Coefficients

10 Appendix E: Covariance Matrices

Table 3: **Dynamic Analysis Covariance Matrices**

Covariance Matrix: 19Jul94 to 10Jan95

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**2.484E-05**	2.385E-05	2.013E-05	1.720E-05	2.382E-05	2.424E-05	3.581E-05	3.861E-05
Arg[Disc]	2.385E-05	**4.090E-05**	2.180E-05	2.041E-05	3.136E-05	3.924E-05	4.110E-05	5.805E-05
Brz[Par]	2.013E-05	2.180E-05	**2.726E-05**	2.117E-05	1.940E-05	2.217E-05	3.624E-05	3.937E-05
Brz[Disc]	1.720E-05	2.041E-05	2.117E-05	**2.137E-05**	1.696E-05	2.051E-05	3.528E-05	3.945E-05
Mex[Par]	2.382E-05	3.136E-05	1.940E-05	1.696E-05	**3.443E-05**	3.565E-05	3.798E-05	5.015E-05
Mex[Disc]	2.424E-05	3.924E-05	2.217E-05	2.051E-05	3.565E-05	**4.480E-05**	4.295E-05	5.886E-05
Vnz[Par]	3.581E-05	4.110E-05	3.624E-05	3.528E-05	3.798E-05	4.295E-05	**9.861E-05**	9.779E-05
Vnz[Disc]	3.861E-05	5.805E-05	3.937E-05	3.945E-05	5.015E-05	5.886E-05	9.779E-05	**1.319E-04**

Covariance Matrix: 11Jan95 to 03Jul95

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.097E-04**	1.197E-04	6.955E-05	6.204E-05	7.339E-05	7.984E-05	8.309E-05	1.024E-04
Arg[Disc]	1.197E-04	**1.489E-04**	7.911E-05	7.414E-05	8.475E-05	9.587E-05	9.973E-05	1.244E-04
Brz[Par]	6.955E-05	7.911E-05	**6.973E-05**	5.282E-05	5.811E-05	6.574E-05	6.835E-05	8.189E-05
Brz[Disc]	6.204E-05	7.414E-05	5.282E-05	**5.977E-05**	5.072E-05	6.114E-05	5.486E-05	7.261E-05
Mex[Par]	7.339E-05	8.475E-05	5.811E-05	5.072E-05	**8.903E-05**	9.227E-05	6.768E-05	8.308E-05
Mex[Disc]	7.984E-05	9.587E-05	6.574E-05	6.114E-05	9.227E-05	**1.090E-04**	7.400E-05	9.622E-05
Vnz[Par]	8.309E-05	9.973E-05	6.835E-05	5.486E-05	6.768E-05	7.400E-05	**1.047E-04**	1.067E-04
Vnz[Disc]	1.024E-04	1.244E-04	8.189E-05	7.261E-05	8.308E-05	9.622E-05	1.067E-04	**1.861E-04**

Covariance Matrix: 05Jul95 to 22Dec95

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.414E-05**	1.284E-05	7.061E-06	6.057E-06	7.275E-06	7.854E-06	7.568E-06	1.201E-05
Arg[Disc]	1.284E-05	**1.420E-05**	6.858E-06	6.685E-06	6.986E-06	8.480E-06	7.542E-06	1.300E-05
Brz[Par]	7.061E-06	6.858E-06	**8.398E-06**	5.452E-06	5.068E-06	5.674E-06	6.340E-06	8.166E-06
Brz[Disc]	6.057E-06	6.685E-06	5.452E-06	**6.146E-06**	4.277E-06	5.283E-06	6.363E-06	9.067E-06
Mex[Par]	7.275E-06	6.986E-06	5.068E-06	4.277E-06	**1.015E-05**	1.029E-05	4.291E-06	7.247E-06
Mex[Disc]	7.854E-06	8.480E-06	5.674E-06	5.283E-06	1.029E-05	**1.271E-05**	5.481E-06	9.260E-06
Vnz[Par]	7.568E-06	7.542E-06	6.340E-06	6.363E-06	4.291E-06	5.481E-06	**2.121E-05**	1.993E-05
Vnz[Disc]	1.201E-05	1.300E-05	8.166E-06	9.067E-06	7.247E-06	9.260E-06	1.993E-05	**3.295E-05**

Covariance Matrix: 26Dec95 to 17Jun96

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.394E-05**	1.460E-05	1.009E-05	9.948E-06	9.388E-06	9.544E-06	1.096E-05	1.335E-05
Arg[Disc]	1.460E-05	**1.667E-05**	1.088E-05	1.100E-05	9.653E-06	1.017E-05	1.115E-05	1.381E-05
Brz[Par]	1.009E-05	1.088E-05	**9.912E-06**	9.313E-06	7.246E-06	7.738E-06	7.955E-06	9.981E-06
Brz[Disc]	9.948E-06	1.100E-05	9.313E-06	**1.011E-05**	7.511E-06	8.446E-06	7.846E-06	1.004E-05
Mex[Par]	9.388E-06	9.653E-06	7.246E-06	7.511E-06	**1.033E-05**	1.018E-05	8.863E-06	1.045E-05
Mex[Disc]	9.544E-06	1.017E-05	7.738E-06	8.446E-06	1.018E-05	**1.137E-05**	8.381E-06	1.038E-05
Vnz[Par]	1.096E-05	1.115E-05	7.955E-06	7.846E-06	8.863E-06	8.381E-06	**1.884E-05**	1.947E-05
Vnz[Disc]	1.335E-05	1.381E-05	9.981E-06	1.004E-05	1.045E-05	1.038E-05	1.947E-05	**3.183E-05**

Covariance Matrix: 18Jun96 to 09Dec96

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**6.080E-06**	5.110E-06	2.758E-06	2.632E-06	2.665E-06	2.489E-06	3.200E-06	3.523E-06
Arg[Disc]	5.110E-06	**5.327E-06**	2.645E-06	2.933E-06	2.533E-06	2.776E-06	2.783E-06	3.665E-06
Brz[Par]	2.758E-06	2.645E-06	**2.432E-06**	2.155E-06	1.673E-06	1.662E-06	1.776E-06	1.953E-06
Brz[Disc]	2.632E-06	2.933E-06	2.155E-06	**2.589E-06**	1.599E-06	1.876E-06	1.624E-06	2.319E-06
Mex[Par]	2.665E-06	2.533E-06	1.673E-06	1.599E-06	**2.448E-06**	2.169E-06	1.905E-06	2.212E-06
Mex[Disc]	2.489E-06	2.776E-06	1.662E-06	1.876E-06	2.169E-06	**2.600E-06**	1.650E-06	2.370E-06
Vnz[Par]	3.200E-06	2.783E-06	1.776E-06	1.624E-06	1.905E-06	1.650E-06	**3.101E-06**	2.711E-06
Vnz[Disc]	3.523E-06	3.665E-06	1.953E-06	2.319E-06	2.212E-06	2.370E-06	2.711E-06	**4.899E-06**

Covariance Matrix: 10Dec96 to 03Jun97

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**3.242E-06**	2.783E-06	1.847E-06	1.940E-06	2.184E-06	2.215E-06	3.010E-06	2.914E-06
Arg[Disc]	2.783E-06	**2.612E-06**	1.627E-06	1.778E-06	1.840E-06	1.968E-06	2.580E-06	2.677E-06
Brz[Par]	1.847E-06	1.627E-06	**1.628E-06**	1.470E-06	1.432E-06	1.359E-06	1.909E-06	1.840E-06
Brz[Disc]	1.940E-06	1.778E-06	1.470E-06	**1.674E-06**	1.442E-06	1.563E-06	2.000E-06	2.155E-06
Mex[Par]	2.184E-06	1.840E-06	1.432E-06	1.442E-06	**2.178E-06**	2.028E-06	2.457E-06	2.206E-06
Mex[Disc]	2.215E-06	1.968E-06	1.359E-06	1.563E-06	2.028E-06	**2.293E-06**	2.458E-06	2.469E-06
Vnz[Par]	3.010E-06	2.580E-06	1.909E-06	2.000E-06	2.457E-06	2.458E-06	**4.407E-06**	3.935E-06
Vnz[Disc]	2.914E-06	2.677E-06	1.840E-06	2.155E-06	2.206E-06	2.469E-06	3.935E-06	**4.440E-06**

Covariance Matrix: 04Jun97 to 24Nov97

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.107E-05**	1.077E-05	9.807E-06	1.012E-05	7.586E-06	6.864E-06	8.268E-06	8.817E-06
Arg[Disc]	1.077E-05	**1.217E-05**	1.103E-05	1.119E-05	7.845E-06	7.243E-06	8.455E-06	9.306E-06
Brz[Par]	9.807E-06	1.103E-05	**1.164E-05**	1.095E-05	7.690E-06	6.831E-06	8.118E-06	9.053E-06
Brz[Disc]	1.012E-05	1.119E-05	1.095E-05	**1.155E-05**	7.540E-06	7.012E-06	8.134E-06	1.005E-05
Mex[Par]	7.586E-06	7.845E-06	7.690E-06	7.540E-06	**6.456E-06**	5.470E-06	6.548E-06	6.913E-06
Mex[Disc]	6.864E-06	7.243E-06	6.831E-06	7.012E-06	5.470E-06	**5.314E-06**	5.745E-06	6.171E-06
Vnz[Par]	8.268E-06	8.455E-06	8.118E-06	8.134E-06	6.548E-06	5.745E-06	**7.597E-06**	7.795E-06
Vnz[Disc]	8.817E-06	9.306E-06	9.053E-06	1.005E-05	6.913E-06	6.171E-06	7.795E-06	**1.130E-05**

Covariance Matrix: 25Nov97 to 19May98

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.800E-06**	1.571E-06	2.093E-06	2.040E-06	1.243E-06	9.555E-07	1.450E-06	1.057E-06
Arg[Disc]	1.571E-06	**1.941E-06**	1.947E-06	2.002E-06	1.096E-06	9.267E-07	1.254E-06	1.076E-06
Brz[Par]	2.093E-06	1.947E-06	**3.220E-06**	2.856E-06	1.616E-06	1.219E-06	1.889E-06	1.405E-06
Brz[Disc]	2.040E-06	2.002E-06	2.856E-06	**3.350E-06**	1.467E-06	1.269E-06	1.771E-06	1.471E-06
Mex[Par]	1.243E-06	1.096E-06	1.616E-06	1.467E-06	**1.413E-06**	9.979E-07	1.122E-06	8.991E-07
Mex[Disc]	9.555E-07	9.267E-07	1.219E-06	1.269E-06	9.979E-07	**1.003E-06**	8.554E-07	7.700E-07
Vnz[Par]	1.450E-06	1.254E-06	1.889E-06	1.771E-06	1.122E-06	8.554E-07	**1.786E-06**	1.185E-06
Vnz[Disc]	1.057E-06	1.076E-06	1.405E-06	1.471E-06	8.991E-07	7.700E-07	1.185E-06	**2.077E-06**

Table 3: (continued) Dynamic Analysis Covariance Matrices (continued)

Covariance Matrix: 20May98 to 09Nov98

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**5.700E-05**	6.382E-05	6.533E-05	8.067E-05	3.574E-05	4.005E-05	6.227E-05	5.181E-05
Arg[Disc]	6.382E-05	**8.243E-05**	6.975E-05	8.782E-05	4.094E-05	4.781E-05	6.502E-05	6.059E-05
Brz[Par]	6.533E-05	6.975E-05	**1.014E-04**	1.156E-04	4.428E-05	4.919E-05	8.982E-05	8.673E-05
Brz[Disc]	8.067E-05	8.782E-05	1.156E-04	**1.494E-04**	5.328E-05	5.609E-05	1.045E-04	9.241E-05
Mex[Par]	3.574E-05	4.094E-05	4.428E-05	5.328E-05	**3.352E-05**	3.284E-05	4.106E-05	3.905E-05
Mex[Disc]	4.005E-05	4.781E-05	4.919E-05	5.609E-05	3.284E-05	**4.460E-05**	4.547E-05	5.724E-05
Vnz[Par]	6.227E-05	6.502E-05	8.982E-05	1.045E-04	4.106E-05	4.547E-05	**1.612E-04**	1.457E-04
Vnz[Disc]	5.181E-05	6.059E-05	8.673E-05	9.241E-05	3.905E-05	5.724E-05	1.457E-04	**2.781E-04**

Covariance Matrix: 10Nov98 to 05May99

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.909E-05**	1.988E-05	3.854E-05	4.335E-05	1.707E-05	1.665E-05	2.022E-05	2.995E-05
Arg[Disc]	1.988E-05	**2.453E-05**	4.184E-05	4.840E-05	1.869E-05	1.862E-05	2.115E-05	3.310E-05
Brz[Par]	3.854E-05	4.184E-05	**1.103E-04**	1.192E-04	3.688E-05	3.615E-05	4.195E-05	6.231E-05
Brz[Disc]	4.335E-05	4.840E-05	1.192E-04	**1.393E-04**	4.148E-05	4.073E-05	4.816E-05	7.117E-05
Mex[Par]	1.707E-05	1.869E-05	3.688E-05	4.148E-05	**2.024E-05**	1.860E-05	2.102E-05	3.059E-05
Mex[Disc]	1.665E-05	1.862E-05	3.615E-05	4.073E-05	1.860E-05	**1.936E-05**	2.115E-05	3.287E-05
Vnz[Par]	2.022E-05	2.115E-05	4.195E-05	4.816E-05	2.102E-05	2.115E-05	**5.766E-05**	6.338E-05
Vnz[Disc]	2.995E-05	3.310E-05	6.231E-05	7.117E-05	3.059E-05	3.287E-05	6.338E-05	**1.266E-04**

Covariance Matrix: 06May99 to 25Oct99

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**1.053E-05**	1.046E-05	7.952E-06	9.628E-06	5.328E-06	5.488E-06	9.926E-06	1.070E-05
Arg[Disc]	1.046E-05	**1.530E-05**	9.592E-06	1.338E-05	6.493E-06	7.590E-06	1.210E-05	1.489E-05
Brz[Par]	7.952E-06	9.592E-06	**1.631E-05**	1.243E-05	6.432E-06	5.635E-06	1.126E-05	1.062E-05
Brz[Disc]	9.628E-06	1.338E-05	1.243E-05	**1.839E-05**	7.517E-06	8.142E-06	1.318E-05	1.563E-05
Mex[Par]	5.328E-06	6.493E-06	6.432E-06	7.517E-06	**5.975E-06**	5.401E-06	7.546E-06	7.577E-06
Mex[Disc]	5.488E-06	7.590E-06	5.635E-06	8.142E-06	5.401E-06	**6.398E-06**	7.786E-06	8.319E-06
Vnz[Par]	9.926E-06	1.210E-05	1.126E-05	1.318E-05	7.546E-06	7.786E-06	**3.214E-05**	2.460E-05
Vnz[Disc]	1.070E-05	1.489E-05	1.062E-05	1.563E-05	7.577E-06	8.319E-06	2.460E-05	**5.153E-05**

Covariance Matrix: 26Oct99 to 14Apr00

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**4.340E-06**	2.802E-06	3.353E-06	1.996E-06	2.066E-06	1.156E-06	3.684E-06	3.960E-06
Arg[Disc]	2.802E-06	**3.376E-06**	2.445E-06	2.215E-06	1.610E-06	1.454E-06	2.430E-06	3.691E-06
Brz[Par]	3.353E-06	2.445E-06	**5.981E-06**	3.458E-06	2.294E-06	1.194E-06	5.202E-06	4.919E-06
Brz[Disc]	1.996E-06	2.215E-06	3.458E-06	**4.009E-06**	1.778E-06	1.557E-06	3.158E-06	4.288E-06
Mex[Par]	2.066E-06	1.610E-06	2.294E-06	1.778E-06	**3.424E-06**	1.939E-06	3.429E-06	2.686E-06
Mex[Disc]	1.156E-06	1.454E-06	1.194E-06	1.557E-06	1.939E-06	**1.990E-06**	1.256E-06	2.032E-06
Vnz[Par]	3.684E-06	2.430E-06	5.202E-06	3.158E-06	3.429E-06	1.256E-06	**1.562E-05**	9.703E-06
Vnz[Disc]	3.960E-06	3.691E-06	4.919E-06	4.288E-06	2.686E-06	2.032E-06	9.703E-06	**1.801E-05**

Table 3: (continued) Dynamic Analysis Covariance Matrices (continued)

Table 4: **Static Analysis Covariance Matrix**

Covariance Matrix: 19Jul94 to 14Apr00

	Arg[Par]	Arg[Disc]	Brz[Par]	Brz[Disc]	Mex[Par]	Mex[Disc]	Vnz[Par]	Vnz[Disc]
Arg[Par]	**2.291E-05**	2.397E-05	1.981E-05	2.055E-05	1.563E-05	1.645E-05	2.075E-05	2.328E-05
Arg[Disc]	2.397E-05	**3.066E-05**	2.158E-05	2.342E-05	1.783E-05	2.021E-05	2.293E-05	2.826E-05
Brz[Par]	1.981E-05	2.158E-05	**3.052E-05**	2.959E-05	1.598E-05	1.703E-05	2.334E-05	2.651E-05
Brz[Disc]	2.055E-05	2.342E-05	2.959E-05	**3.544E-05**	1.625E-05	1.776E-05	2.384E-05	2.753E-05
Mex[Par]	1.563E-05	1.783E-05	1.598E-05	1.625E-05	**1.828E-05**	1.816E-05	1.700E-05	2.034E-05
Mex[Disc]	1.645E-05	2.021E-05	1.703E-05	1.776E-05	1.816E-05	**2.181E-05**	1.813E-05	2.402E-05
Vnz[Par]	2.075E-05	2.293E-05	2.334E-05	2.384E-05	1.700E-05	1.813E-05	**4.380E-05**	4.194E-05
Vnz[Disc]	2.328E-05	2.826E-05	2.651E-05	2.753E-05	2.034E-05	2.402E-05	4.194E-05	**7.332E-05**