Recent Advances in Modeling Liquidity Risk and Applications to Central Clearing

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Outline

• Liquidity in risk management: listed markets, OTC Markets
• Liquidity-adjusted VaR for directional exposures
• BM&F Bovespa’s Close-out Risk Evaluation (CORE)
• Modeling Liquidity Charges in OTC Markets & Liquidity Polls
Importance of Liquidity Modeling

• Need to go beyond VaR. In most cases, VaR is grossly insufficient due to prolonged, expensive liquidation.

• LTCM (1998): basis trades (long high spreads/short low spreads, DV01=0)

• Credit Crunch (2007, 2008): loss of liquidity for financing MBS, ABS and Credit Derivatives

• J.P. Morgan CIO (2012): basis trade, index CDS versus corporate CDS (2bb loss -> 8 bb loss)

• MF Global (2012): 2y term repos on Italian government bonds

• Grupo Interbolsa (2012): Fabricato share repos, collapse of 2nd largest Colombian broker-dealer
Main Themes in Liquidity Modeling

Liquidity = the ability to convert a given position into cash or risk-less securities

- **Size matters:**
  \[ \Delta P = Const. \times |Q|^p \]
  The larger the quantity, the worst is the outcome. \( p=1.5 \) is generally accepted, but there exists some controversy among academics, e.g. Gabaix \( p=0.5 \), Cont \( p=1 \)

- **Time-horizon matters:** there is a maximum reasonable amount of inventory that can be liquidated in a given period of time (Almgren & Chriss, 2000).
  Break large orders into smaller pieces (TWAP, WVAP, PoV)

- **Market structure matters:** OTC derivatives markets have different price/liquidity discovery properties than listed markets.
Differences between OTC and Listed Markets

- In **exchanges**, liquidity models are based on daily trading volumes, open interest and bid-ask spreads.

- In **over-the-counter (OTC)** markets, trades and volumes are unknown and post-trade/market depth data are not easy to find (Avellaneda and Cont, 2010).

- **Main challenge**: to calculate suitable liquidity reserves or ```charges``` for portfolios.

- This problem is directly relevant to **central counterparty clearing** of derivatives and portfolio risk-requirements in the post-2008 era. In CCPs, liquidity must be modeled explicitly.
Liquidity-adjusted VaR: simple exercise or complex calculation?

Initial jump modeled by Var/ES/CVaR (``market risk'')

Large size implies further loss due to market impact
Heuristics: Liquidity Curves

\[ L(Q) = a \times |Q|^{1.5} \]

Good for ``outright” positions.
Not good for portfolios, where we have offsets between long and short positions in instruments with common RFs.

\[ L\text{VaR}(Q \cdot X) = V\text{aR}(Q \cdot X) + a|Q|^{1.5} \]
Simple liquidity model

\[ Q = \text{notional quantity} \]
\[ Q_0 = \text{``typical'' trade size} \]
\[ \sigma = \text{price volatility (\%)} \]
\[ N = Q/Q_0 \text{ dimensionless size of the position} \]
\[ \xi_i = \text{iid Student-t r.v. mean=0, variance=1, df >2} \]

\[ PNL(n) = \sum_{i=1}^{n} \sigma (Q - iQ_0) \xi_i \]
\[ = \sigma Q \sqrt{N} \sum_{i=1}^{n} \left(1 - \frac{i}{N}\right) \frac{\xi_i}{\sqrt{N}} \]
\[ = \sigma Q_0 \left(\frac{Q}{Q_0}\right)^{1.5} \sum_{i=1}^{n} \left(1 - \frac{i}{N}\right) \frac{\xi_i}{\sqrt{N}} \]
\[ \approx \sigma Q_0 \left(\frac{Q}{Q_0}\right)^{1.5} \int_0^\tau (1 - t) dW_t \quad N, n \gg 1, n/N \to \tau \]
Liquidity Add-on?

- Scaling gives a PNL for liquidation which is a standardized *stochastic process* (stochastic integral) multiplied by a nonlinear function of quantity.

- Any risk-measure (e.g. VaR, CVaR, STDEV) will give rise to a liquidation cost of the form

\[
PNL = \kappa \sigma Q_0 \left( \frac{Q}{Q_0} \right)^{1.5} = \kappa \sigma \frac{Q^{1.5}}{Q_0^{0.5}}
\]

- This is the asymptotics for large Q. Therefore, the LC should be something like this

\[
LC(Q) = \kappa \sigma Q_0 \max \left[ \left( \frac{Q}{Q_0} \right)^{1.5}, \frac{Q}{Q_0} \right] \quad \kappa \sigma \approx \frac{1}{2} \text{ (bid-ask spread)}
\]
Example: Eurodollar Futures

- CME ED Futures: Median Volume ~ 100 K contracts
- Tick size (and bid/ask spread) = 0.25 bps for front month, 0.5 bps others.
- Contract sensitivity = USD 25 per LIBOR basis point move
- Reasonable volume = 10% daily volume = 10,000 contracts
- Typical liquid (large) trade = 10,000 × 25 = 250,000 USD of DV01
Deriving Liquidity Curves

- Liquidity Charge for normal trades = ½ bid-ask spread

Front month = 1/8 basis point
Out months = ¼ basis point

\[ LC(X) = \frac{1}{2} s \times \left( \frac{X}{X_0} \right)^{1.5} \]

\[ LC_f(X) = (0.125) \left( \frac{X}{0.25} \right)^{1.5} = X^{1.5} \]

\[ LC_b(X) = (0.25) \left( \frac{X}{0.25} \right)^{1.5} = 2X^{1.5} \]

<table>
<thead>
<tr>
<th>spread</th>
<th>1 MM</th>
<th>5 MM</th>
<th>10 MM</th>
<th>25 MM</th>
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<td>11</td>
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<tr>
<td>0.5</td>
<td>2</td>
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<td>250</td>
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<tr>
<td>1</td>
<td>4</td>
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<td>126</td>
<td>500</td>
</tr>
</tbody>
</table>

Liquidity charges in basis points of notional
For different sizes measured in MM Dv01
Market and Liquidity Risk for Portfolios
Central clearing & risk-management

Large circles: Clearing members (large banks, BDs)

Small circles: non-clearing market participants (buy side)

Arrows represent credit exposure
Major Clearinghouses Today

Inter-bank payments: ACH

Securities: DTCC, FICC, LCH.Clearnet, Eurex Clearing

Derivatives: CME Group, LCH.Clearnet, Intercontinental Exchange (ICE), ICE Clear Europe, Eurex Clearing, Hong-Kong Exchanges and Clearing

Equity Options: The Options Clearing Corporation

In Brazil, BM&F Bovespa manages 4 CCPs for different asset classes (like CME Group, which clears commodities, financials, and some OTC)
Tools for Risk Management of CCPs

- Initial Margin (Market Risk, **Liquidity Risk**)
- Fund for mutualization of losses (``Guarantee Fund’’), covering shortfall beyond the IM
- ``Loss tranches’’ with CCP’s own capital (Skin in the game)

BIS, Principles for Financial Markets Infrastructures, BIS CPSS-IOSCO Consultative Report, April 2012

ESMA, Final Report, Technical Standards on OTC Derivatives, CCPs and Trade Repositories (``EMIR’’), September 2012
Some CCP Risk-management issues which combine market and liquidity risk

- How can the CCP remain well-capitalized during the liquidation of a defaulted participant?
- Create synergies by using a common system to clear different products with the same risk factors (e.g. listed and OTC derivatives, collateral-in-margin)?
- How to handle a portfolio of securities sensitive to the same risk-factors but having different liquidity?
- Treat portfolios which have daily settlement (futures) as well as OTC securities (forwards) which do not have daily settlement?
Close Out Risk Evaluation (CORE)
**Close-Out Risk Evaluation (CORE) proposed by BM&F Bovespa**

- Find a suitable liquidation strategy for each participant’s portfolio
- Compute potential uncollateralized losses associated with liquidation of the portfolio under stress scenarios
- Margin requirement is based on potential losses over the liquidation period

Example 1: Liquid vs. illiquid stock

Petrobras average trading volumes
PETR4: 24 million shares/day (assume max liq. = 3MM)
PETR3: 8 million shares/day (" = 1MM)

Portfolio: Long 40 MM PETR4, short 30MM PETR3

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>5</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETR4 (sell)</td>
<td>3 3 3 3 1 1 1 1 1 1 1 1 1</td>
<td></td>
<td>1 1 1 1 1 1</td>
</tr>
<tr>
<td>PETR3 (buy)</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td></td>
<td>1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

Mkt. exposure: 10 MM PETR4 for 5 days
PETR 4 "hedges" PETR 3 MTM P/L during liquidation
Liquidating independently of common risk factors (naïve liquidation)

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>5</th>
<th>14</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETR4 (sell)</td>
<td>3 3 3 3 3 3 3 3 3 3 3 1</td>
<td>0 0</td>
<td>0------------------------</td>
<td>0 0 0</td>
</tr>
<tr>
<td>PETR3 (buy)</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1</td>
<td>------------------------</td>
<td>1 1 1</td>
</tr>
</tbody>
</table>

Mkt. exposure: 26 MM PETR4 for first 13 days, 16 MM PETR3 for last 15 days
Much more market risk than previous example.

Naïve liquidation always costs more. Some optimization can be done.
Example 2: portfolio of futures and OTC forwards with bond collateral

Portfolio:

- Long 10,000,000 BRL in USD futures (liquid)
- Short 10,000,000 BRL in forwards (illiquid, with auction in 10 days)
- Long 20,000,000 in Brazilian T-bills (LTN) (liquid, but not cash)

Naïve Strategy #1:

- Close futures position, sell T-bills, wait 10 days with the forward position

Naïve Strategy #2:

- Do not close futures position, wait 10 days with forward position, then sell T-bills
Correct strategy takes into account daily settlement of futures

Best strategy:

• Do not close futures position, wait 10 days with forward position,

Sell a fraction of T-Bills to cover variation margin in Futures for 10 days. Close the remaining portfolio in 10 days

Naïve #1

Naïve #2

(may require more collateral)

Best strategy
Modeling portfolios with liquidity constraints

• In a world with infinite liquidity, a portfolio is represented as a list of instruments and quantities

```
<table>
<thead>
<tr>
<th>DOL Fut 01/2013</th>
<th>VALE5</th>
<th>GUAR3</th>
<th>BOVA11</th>
<th>IBOV Fut 04/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>-45,000</td>
<td>53,000</td>
<td>-20,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>
```

• In a world with limited liquidity, we should include the maximum amounts that can be traded in a given period (day) without `moving the market’*

```
<table>
<thead>
<tr>
<th>DOL Fut 01/2013</th>
<th>VALE5</th>
<th>GUAR3**</th>
<th>BOVA11</th>
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<tr>
<td>2,000</td>
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<tr>
<td>25,000</td>
<td>1,000,000</td>
<td>1,000</td>
<td>150,000</td>
<td>10,000</td>
</tr>
</tbody>
</table>
```

* Proxied here at 10 % Avg. Traded Volume
** Guararapes Confecc. SA
Portfolio Description

- \( R \) represents the state of the market or path of states of the market (risk-factor changes)

\[
R = (R_0, R_1, R_2, \ldots R_t, R_{t+1}, \ldots)
\]

- Example: if we are dealing with options, then

\[
R_t = \begin{pmatrix}
S_t \\
\sigma_t \\
r_t \\
d_t
\end{pmatrix}
\]

- \( Q_i, l_i \) represent quantities and daily liquidity limits for each instrument.

<table>
<thead>
<tr>
<th>( MTM_1(t,R) )</th>
<th>( MTM_2(t,R) )</th>
<th>( MTM_3(t,R) )</th>
<th>( MTM_4(t,R) )</th>
<th>( MTM_5(t,R) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 )</td>
<td>( Q_2 )</td>
<td>( Q_3 )</td>
<td>( Q_4 )</td>
<td>( Q_5 )</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>( l_2 )</td>
<td>( l_3 )</td>
<td>( l_4 )</td>
<td>( l_5 )</td>
</tr>
</tbody>
</table>
Liquidation of a Portfolio: `Close-out strategy’

- On date \( t=0 \), you decide that a portfolio should be liquidated starting on \( t=1 \).

- Determine a strategy in which a certain fraction, \( q_{it} \), of the position in instrument \( i \) will be liquidated at date \( t \). ( \( q_{it}, i = 1, ..., N, t = 1, ..., T_{max} \))

\[
0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \quad \forall i \forall t
\]

\[
\sum_{t=1}^{T_{max}} q_{it} = 1
\]

\[
n_t = \sum_{s=t+1}^{T_{max}} q_s
\]

- A close-out strategy is a matrix that tells us how to proceed for liquidating the various instruments in the portfolio as time passes.

The remaining balance (%) at time \( t \)
Defining the objective function: Profit and loss of a close-out strategy for a portfolio

\[ \psi_i(t, R_t) \overset{\text{def}}{=} Q_i[MTM_i(t, R_t) - MTM_i(0, R_0)] \]

**Realized** P/L at date \( t \), after trading

\[ L_r(t, q, R_t) = \sum_{i=1}^{N} q_{it} \psi_i(t, R_t) \]

**Unrealized** (a.k.a. MTM) P/L at date \( t \), after trading

\[ L_{nr}(t, q, R_t) = \sum_{i=1}^{N} n_{it} \psi_i(t, R_t) \]
Accumulated P/L

- Accumulated profit/Loss for close out strategy at date $t$

\[
L(t, q, R) = \sum_{s=1}^{t} L_r(s, q, R_s) + L_{nr}(t, q, R_t)
\]

\[
= \sum_{s=1}^{t} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t)
\]

- cash
- unrealized gain/loss
CORE objective function

- Define scenarios for the risk-factors, \( R = (R_1, R_2, \ldots, R_{T_{max}}) \)

- These scenarios are paths. Let \( R \) denote the set of all scenarios considered

\[
U(q) = \min_{R \in R} \min_{1 \leq t \leq T_{max}} L(t, q, R)
\]

\[
= \min_{R \in R} \min_{1 \leq t \leq T_{max}} \left( \sum_{s=1}^{t} \sum_{i=1}^{N} q_{is} \psi_i(s, R_s) + \sum_{i=1}^{N} n_{it} \psi_i(t, R_t) \right)
\]
The Optimization Problem

Maximize \( U(q) \quad q = (q_{it}) \in \mathbb{R}^{N \times T_{\text{max}}} \)

Subject to:

\[
\begin{align*}
0 & \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \quad \forall i \quad \forall t \\
T_{\text{max}} \sum_{t=1}^{T_{\text{max}}} q_{it} & = 1 \quad \forall i
\end{align*}
\]

- \( U(q) \) is a sum of minima of linear functions of \( q \) \( \Rightarrow \) it is concave
- The set of constraints is convex (it is a convex polyhedral region)

A solution exists and should be unique under reasonable conditions!
Solution Via Linear Programming for $U_2$

Maximize: $U$

Variables: $\{U, \lambda_t, \mu_t, q_{it}; 1 \leq t \leq T_{max}, 1 \leq i \leq N\}$

Subject to constraints:

- $U \leq \lambda_t + \mu_t \quad \forall t$
- $\lambda_t \leq \sum_{i=1}^{N} q_{it} \psi_i(t, R_t) \quad \forall t \forall R \in R$
- $\mu_t \leq \sum_{i=1}^{N} (q_{it} + n_{it}) \psi_i(t, R_t) \quad \forall t \forall R \in R$
- $0 \leq q_{it} \leq \frac{l_i}{Q_i} \equiv k_i \quad \forall i \forall t$
- $\sum_{t=1}^{T_{max}} q_{it} = 1 \quad \forall i$
BMF-CORE Liquidity Adjusted Risk Margin

\[
M = \max_q \min_{R \in R} \left( \min_{1 \leq t \leq T_{\text{max}}} L(t, q, R) \right)
\]

\[
= \min_{R \in R} \left( \min_{1 \leq t \leq T_{\text{max}}} L(t, q^*, R) \right)
\]

\[q^* = \text{optimal close-out strategy}\]

- **Alternative versions**, which can be used with Monte Carlo models for RFs,

\[
\text{VaR}_\alpha \left( \min_{1 \leq t \leq T_{\text{max}}} L(t, q^*, R) \right) \quad \alpha = .99, \text{or} .995
\]

\[
\text{ES}_\alpha \left( \min_{1 \leq t \leq T_{\text{max}}} L(t, q^*, R) \right) \quad \alpha = .99 \ldots
\]
Example: Liquidation of a portfolio of stocks using the CORE risk-measure and Historical Monte Carlo

- 10 stocks, 100 shares per stock

| SPY | GDX | UVXY | VTI | VWO | SIL | TLT | IWM | AGG | VOO |

$$ES(X,T) = ES_{.99} \left( \min_{t \leq T} L(t, q, R) \right)$$

Strategy q: liquidate equal amounts of stocks each day (not optimal)
Portfolio Liquidity Modeling in OTC Markets
Liquidity in OTC Risk-Management

- In OTC risk-management, the portfolio is liquidated in an auction (there is not exchange).

- Participants periodically inform the CCP on liquidity and market depth, so IM requirements can take into account liquidation costs.

- Difficulties may arise since the ex-ante liquidation costs used for risk management are provided by agents which will bid on the defaulted portfolio at auction (ex-post).

- The time dimension of liquidation should be ``made equivalent’’ to a wider B/O spread in a 1-day auction.
Polling

• Polls are conducted asking CCP participants by how much would their bid or ask price change as a function of trade size

• Liquidity polls typically involve
  -- directional positions
  -- market-neutral portfolios

• Liquidity charges for market-neutral portfolios are typically lower than for outright positions because they have less risk exposure

• To some extent, the poll incorporates the time-dimension of the close-out process
Example: Swaps

<table>
<thead>
<tr>
<th>Tenor (yrs)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port 1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Port 2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Port 3</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Port 4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Port 5</td>
<td>1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Port 6</td>
<td>1</td>
<td></td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>Port 7</td>
<td>1</td>
<td></td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>Port 8</td>
<td>1</td>
<td></td>
<td>-1</td>
<td></td>
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<tr>
<td>Port 9</td>
<td></td>
<td></td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Port 10</td>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>Port 11</td>
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<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>Port 12</td>
<td>1</td>
<td>-2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Port 13</td>
<td>1</td>
<td>-2</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

- We consider 4 standard swap tenors. A typical poll will consider several portfolios: outright swap positions, curve positions (or time spreads) and butterfly spreads. **Portfolios 5 to 13 are market-neutral.**
Liquidity Charge Curves obtained by polling 10 dealers and taking median values

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>multiplier (1X, 5X, 10X, 25X)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.25</td>
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<tr>
<td>12</td>
<td>4.00</td>
</tr>
<tr>
<td>13</td>
<td>4.00</td>
</tr>
</tbody>
</table>

- We use the median value as an indicator of the function F(N) for each spread
- Rows 1 and 2 are similar to what we obtained earlier for ED futures based on 1.5 model
Smoothing the Poll Data

- Take the discrete poll and fit the data to power-laws using log-log regression

\[ F(N) = (1.26827) \times N^{1.6406} \]

Bps for 1MM DV01

Exponent greater than 1
Empirical results for LCCs

<table>
<thead>
<tr>
<th>Spread</th>
<th>ln a</th>
<th>a</th>
<th>b</th>
<th>R-squared</th>
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</thead>
<tbody>
<tr>
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<td>1.524495236</td>
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</tr>
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<td>1.616243426</td>
<td>0.989358386</td>
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<tr>
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<td>1.548779256</td>
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</tr>
<tr>
<td>13</td>
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<td>3.882009196</td>
<td>1.511079491</td>
<td>0.984376</td>
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</tbody>
</table>

Table 1: Results of log-log regression for the 13 spreads in the swaps example, using median poll values. The fit is to the curve $F = aN^b$ which is done by estimating the linear model $lnF = lna + b(lnN) + \epsilon$. The $R^2$ coefficient is given in the right-most column. Notice that the empirically estimated exponents $b$ are reasonably close to the $b=1.5$ heuristic value.
Liquidity Add on
Charge Calculation for IR Swap Portfolios

• Represent swap portfolios as loadings on the standard tenor swaps (2y,5y,10y,30y)

• Minimize:

\[ \sum_{i=1}^{N} F_i(Q_i) \]

subject to the linear constraints

\[ \sum_{i=1}^{N} \mu_{im} Q_i = \Delta_m \quad m = 1, \ldots, M \]

and bound constraints

\[ |Q_i| \leq Q_{\text{max},i} \quad i = 1, \ldots, N \]
Example

**TARGET PORTFOLIO**

<table>
<thead>
<tr>
<th>TENOR</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>30</th>
</tr>
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<tbody>
<tr>
<td>DV01 MM</td>
<td>12</td>
<td>-18</td>
<td>5</td>
<td>-5</td>
</tr>
<tr>
<td>CHARGE (bp)</td>
<td>74.78</td>
<td>155.15</td>
<td>22.01</td>
<td>34.26</td>
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<tr>
<td>NAÏVE CHG</td>
<td>286.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CALCULATED SPREAD</th>
<th>HEDGE POSITION</th>
<th>LIQUIDITY</th>
<th>CHARGE</th>
</tr>
</thead>
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<tr>
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<td>0.38</td>
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<tr>
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<tr>
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<td>0.00</td>
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<td>13</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Sum of changes: 167.15

NAÏVE CHARGE: 286.20

SMART CHARGE: 167.15

Exp Shortfall 99%: 234,136,074 (Margin)

Liquidity Charge: 167,145,075
Conclusions

- Liquidity Modeling is an integral part of risk management.

- In CCPs, liquidity is essential for constructing a sound margin system for portfolios.

- Models should include portfolio size and time-horizon (model the close-out!)

- CORE (BMF&F Bovespa) suggests constructing liquidity thresholds for each security and finding a liquidation strategy so that the worst value of the portfolio along the liquidation period under stress scenarios is optimized.

- In OTC markets, where prices are contributed by participants, liquidity polls for reference portfolios at different trade sizes can be used to build liquidity curves.

- There is some indication that poll-based estimates are consistent with the CORE approach. Empirically we found that LCCs from polls are convex functions of trade size and appear to follow the “1.5 model”.