Derivative Securities: Homework 2

1. A four-month European call option on a dividend-paying stock is currently selling for $5.00. The stock price is $64.00, the strike price is $60, and a dividend of $0.80 is expected in one month. The risk-free rate is 12% per annum. What opportunities are there for an arbitrageur?

2. Let XYZ be a dividend-paying stock which goes ex-dividend on date T_{ex}. Show that if an option expires after T_{ex} it may be optimal to exercise the option, but if so, only immediately prior to the ex-dividend date.

3. Justify the Put-Call parity formula for American-style options:

$$C_{pop} - P_{pop} = KRT - SDT + C_{eep} - P_{eep} \tag{1}$$

where C_{pop}, P_{pop} are premia over parity for calls and puts, K is the strike price, R is the (simply compounded) interest rate, T is time to maturity measured in years,S is the underlying spot price, D is the simply compounded dividend yield and C_{eep}, P_{eep} denote the early exercise premia for calls and puts. By definition, the *early-exercise premium* is the difference between the value of an option and the theoretical value of the option if held to maturity.

How does this formula relate to the Put/Call parity formula for in Hull? [Formula (7.3), p. 174, in 4th edition].

4. Given a stock and an option expiration date, we define the pseudo-dividend yield, $\tilde{D} = \tilde{D}(K,T)$, by the formula

$$\tilde{D} = \frac{P_{pop} - C_{pop} + KRT}{ST}.$$

Using the data provided in the accompanying spreadsheet, compute the pseudo-dividend yields for IBM options for all the strikes and expirations until January 2010. Assume that the interest rate is 2.60% (flat). Graph the pseudo-dividend function for each of the option series. Using the actual dividend stream (and reasonable forecasts for future dividends) compute the actual dividend yields D of IBM for different expirations. Show that

$$\tilde{D} = D + \frac{P_{eep} - C_{eep}}{ST}$$

or

$$P_{eep} - C_{eep} = ST\left(\tilde{D} - D\right).$$

Plot curves of $P_{eep} - C_{eep}$ for each expiration date as a function of strikes. Show that $P_{eep} - C_{eep}$ is approximately zero near-the-money. Justify your answer. Explain the shape/behavior of the pseudo-dividend curve for a given maturity.

5. Consider the option market for VMWARE Inc. (VMW) on Feb 29, 2008 in the attached spread-sheet. VWM pays no dividends and is a hard-to-borrow stock (subject to Regulation SHO).

For each of the expirations given in the data, calculate the pseudo-dividend yield $\tilde{D}$ as a function of strike. If we assume that $P_{eep} - C_{eep}$ vanishes for ATM options (see above exercise), show that

$$\tilde{D} \; = \; D + \frac{P_{eep} - C_{eep}}{ST},$$

where D is an *effective* dividend yield that makes Put-Call Parity (1) hold. Give the numerical value of D for each of the expirations in the data. Since VMW does not pay dividends, conclude that Put-Call-Parity does not necessarily hold for options on assets that are hard-to borrow. Support your answer by an arbitrage-type argument. Can you give a financial interpretation for D? Verify your conclusions by repeating the analysis for DNDN (Dendreon Corp.), which is another hard-to-borrow stock.

6. Consider three European call options with strikes $K_1 < K_2 < K_3$ and respective mid-market prices C_1, C_2, C_3. Show that the inequality

$$C_2 \; \leq \frac{K_3 - K_2}{K_3 - K_1} C_1 + \frac{K_2 - K_1}{K_3 - K_1} C_3$$

to within bid-ask spread errors, based on an arbitrage argument. [Hint: Construct a portfolio of the three options that has a non-negative payoff,which is positive on the interval (K_1, K_3).]