TOPOLOGY I

The Vicious Dog (An Introduction to Classifying Surfaces)

Background:

Topology is a fundamental branch of mathematics. Part of it relates to geometry. When a topologist studies geometric objects, he or she is concerned with only certain properties of those objects -- the properties that remain unchanged as the object is stretched, twisted, and distorted. (The exact mathematical term is 'continuously deformed'.)

For example, consider what remains the same as object A is distorted into object B, below.

Shapes can be radically distorted. Distances and angle measures change in obvious ways. But what remains the same? Objects A and B have the same number of points and arcs connecting them. They also show the same number of 'patches,' or regions, and these regions border each other in the same way. The properties that survive the distortion (such as how regions border each other) are called "topological invariants." When an object can be distorted into another as in the case above, we say they are "topologically equivalent," or "homeomorphic."

Classifying surfaces:

One of the most fundamental issues in topology is how to classify surfaces. Points, lines and shapes "behave" differently on different kinds of surfaces. So a topologist needs to know what kind of surface he/she is dealing with.

The first step is to see that some shapes are fundamentally different from others, and to do that we begin, not with a discussion of surfaces and holes, but instead with the story of a vicious dog and a fence.
Imagine that you are in a grassy field with a vicious dog, and the only thing you have for protection is a fence, which forms a closed loop around the dog (see below). Where would you like to stand? (This is not a trick question. It IS that easy, but it gets harder.)

Of course, you would like to stand outside the fence.

Now suppose you live on the surface of a sphere. (In fact we do!) Does your answer change? How about the surface of a cube? A potato?

It gets harder to distinguish the "inside" from the "outside" at some point, but with all of these surfaces there is a safe place to stand - just on the other side of the fence from the dog.
Now consider the surface of a doughnut. (This important shape is called a torus), and suppose the fence is drawn like this:

Exercise 1: For each example of a fence and a dog (on a torus), is there a safe place to stand?
On the surface of a torus we have to be more careful. Sometimes even if there is a closed loop, there is no safe place to stand. Sometimes there is no "inside" and "outside" of the fence.